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Novel Optimum Design and Sensitivity Assessment for Concrete Beams Using FRP Bars

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Abstract

Optimization of structural elements is one of the most important topics in structural engineering, with a wide range of applications, including concrete beams reinforced with fiber-reinforced polymer (FRP) bars. The increasing use of FRP reinforcement, owing to its corrosion resistance and high strength-to-weight ratio, has created a need for efficient and economical design approaches. Therefore, the objective of this paper is to employ the Lagrangian Multiplier Method (LMM) to obtain a minimum-cost design of singly reinforced concrete beams with rectangular cross-sections. In this study, the costs of concrete and FRP materials are formulated as objective cost functions to be minimized, while the ultimate flexural strength of the beam is considered the primary design constraint. The optimization problem is solved analytically, and optimum designs are derived in closed-form solutions using the LMM framework. The applicability of the proposed formulation is demonstrated through a real-life design example of a singly reinforced concrete beam, showing that the minimum-cost design can be effectively achieved while satisfying structural performance requirements. The derived design relations and optimization curves provide a practical and efficient tool for engineers seeking economical and reliable FRP-reinforced concrete beam designs.

Keywords: Composite Reinforced Concrete Beam, Optimization, Design Curves, FRP Bar, Lagrangian Multiplier Method, ACI

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1. Introduction

Plain concrete members are strong in compression, but they cannot bear tension stresses due to their natural characteristics. Thus, at the very beginning, it was used for simple, massive structures. Later, designers and builders developed techniques to embed bars into concrete members so as to provide additional capacity to resist tensile stresses. This revolutionary effort ended with what is known as reinforced concrete (RC). Steel bars were the only option for reinforcing concrete structures. Later, with the advent of fiber-reinforced polymer (FRP) replacing steel reinforcement to achieve higher performance in reinforced concrete members, it was found that FRP is remarkably beneficial (Bakis et al., 2002). Corrosion of steel reinforcements in intensive environments can cause serious damage in RC structures(Fernandez et al., 2015). To prevent damage, the use of FRP bars as flexural reinforcement was proposed. Due to the non-corrosive property, the use of FRP bars can reduce protection and rehabilitation costs(ACI Committee 440., 2015; Hollaway, 2010). Researchers in concrete engineering have recently realized that it is time to present design optimization methods for such applications (Adamu et al., 1994; Adamu & Karihaloo, 1994). The goal of structural optimization is to make the best or most effective use of materials for structural elements that can satisfy design requirements, which are usually to reduce the design costs of materials. With accurate implementation of detailing and quality control, concrete structures have a much more overtaking position in terms of weight, strength, and ductility compared to metal-framed structures(Ozbay et al., 2010). Optimization design plays a key role in the economic design of reinforced concrete elements. The optimal section economic designs carried out low reinforcement rates owing to high cost of reinforcing bars (Bordignon & Kripka, 2012; Saini et al., 2007). Beams as structural components are of the main parameters that implementation on their optimized design can reduce the overall cost of the structure (Ceranic & Fryer, 2000). Beam component costs are significantly controlled based on their dimensions, ratio of reinforcements, and costs per unit(Saini a et al., 2006) . The objective of optimization (e.g., minimizing the cost, weight, etc.) thus, the range of design variables and constraints are widely presented in recent literatures with different optimization methods to acquire an optimized design (Rahmanian et al., 2014).

Designing structural elements requires adjudication, insight, experience, and the ability to design structural elements at safe levels concerning serviceability and economy. Design codes and codes of practice do not implement the aforementioned parameters. So, the designers have to perform multiple design cycles to find the best solution. It is relatively difficult to achieve the minimum design cost of RC beams using conventional methods, while there is a large number of design solutions that yield equal bending moment capacity (Ferhat Fedghouche & Tiliouine, 2012). Thus, implementing a numerical optimization technique becomes necessary to develop a cost-effective design approach (Al-Salloum & Husainsiddiqi, 1994; F. Fedghouche & Tiliouine, 2010; Ozturk et al., 2012). The optimization includes selecting the design parameters concerning the overall cost of the beam to be minimized, by which performance and geometrical constraints are satisfied. The prerequisite to the design of reinforced concrete beams is to limit geometrical dimensions because of architectural considerations and the load applied to the beam, which is greater than the limited moment capacity in beams with a doubly reinforced system compared with singly reinforced concrete beams(Saini et al., 2007). An attempt to minimize the weight of structures by controlling the compression steel in concrete structures has been made by researchers (Atabay, 2009), whereas most researchers are acquiring techniques to reach an optimized cost of the structures(Öztürk et al., 2016). It is obvious that the weight of the structure is proportional to the cost of materials; thus, minimizing the use of materials should be the main objective in designing RC structures. Many researchers have used the ultimate load method to design concrete elements(Chakrabarty, 1992; A. Mukherjee & Deshpande, 1995a; Abhijit Mukherjee & Deshpande, 1995), whereas not many employed the limit state method (Al-Salloum & Husainsiddiqi, 1994; Ceranic & Fryer, 2000). To meet constraint satisfaction criteria, researchers implemented methods to satisfy moment capacity constraints, others considered the weight of the structure in their analyses(Chakrabarty, 1992; Ferhat Fedghouche & Tiliouine, 2012; A. Mukherjee & Deshpande, 1995b), and likewise, a few researchers have given design equivalents related to the deflection by considering factored loads (Adamu et al., 1994; Adamu & Karihaloo, 1994).

Optimization techniques can be categorized into three main fields: mathematical programming, methods based on optimality criteria, and heuristic search algorithms (Ferhat Fedghouche & Tiliouine, 2012). A few researchers have applied heuristic search algorithms such as artificial bee colony, simulated annealing, and artificial neural networks for acquiring the optimum design of reinforced concrete beams(de Medeiros & Kripka, 2013; Kao & Yeh, 2014; Ozturk et al., 2012). Lagrangian multiplier methods have been successfully applied in optimizing constrained problems in engineering(Arora et al., 1995). These methods perform direct transformation of constrained problems to unconstrained problems, resulting in a solution through a system of sequential unconstrained optimization subordinate problems. This approach has been employed for minimizing the cost design of singly reinforced concrete beams with rectangular shapes to resist the external action of flexural bending based on the British Standard(Ceranic & Fryer, 2000). Creativity of researchers in optimization engineering has led to combination of Lagrangian Multiplier Method with other optimization approaches. For instance, an application of the Continuum-type Optimality Criteria (COC) method to the design of RC beams, where the conditions of minimality are derived using the augmented Lagrangian method, has been proposed, where the cost that is minimized consists of concrete, reinforcement, and formwork costs with active constraints on maximum deflection, bending, and shear strength(Adamu & Karihaloo, 1994). Thus, the objective of this paper is to employ LMM to acquire a minimum cost design of singly reinforced concrete beams with rectangular shapes based on ACI-318-14 and ACI-440-15 codes. Concrete and FRP material costs are used as objective cost functions to be minimized, and the ultimate flexural strength of the beam is considered to be the main constraint. The optimum design curves achieved in this study can be used for minimum cost design of the beams without prior knowledge of optimization. The results were presented in closed-form equations, which can be easily used by scientists and engineers without the need for performing the design process.

2. Concrete Properties

Several models representing the behavior of concrete in compression are available. The compressive stress–strain diagram for normal strength concrete proposed by (Todeschini, Bianchini, and Kesler) is represented in Figure 1. f'_c is the design concrete compressive strength, and ε_{cu} is the maximum usable concrete compressive strain and is assumed equal to 0.003.

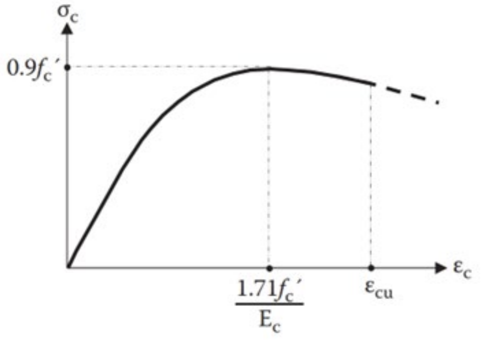


Figure 1. Concrete compressive strength

For ultimate strength calculations which controlled by concrete crushing, ACI 318-14(ACI, 2014) allows the approximation of the stress–strain curve to an equivalent rectangular stress, or "stress block," distribution as discussed in Section 4.5. In the design examples discussed in the chapters of Part 3, the stress–strain curve proposed by (Todeschini et al., 1964) is adopted when concrete crushing does not control failure.

Modulus of elasticity of concrete. The modulus of elasticity of concrete is dependent to concrete compressive strength (f'_c), concrete age, properties of cement and aggregates, and rate of loading. Based on statistical analysis of experimental data available for concrete with unit weights, w, varying between 90 and 155 pcf (1442 and 2483 kg/m3), ACI 318-14 provides the following empirical equation for computing the modulus of elasticity:

$$E_{concrete} = 0.43\, w^{1.5}\sqrt{f_c'} \quad (1)$$

For normal-weight concrete weighing 145 pcf (2323 kg/m3), the following simplified equation is suggested by ACI 318-14:

$$E_{concrete} = 4700\sqrt{f_c'} \quad (2)$$

3. Minimum FRP Reinforcement

ACI 440.1R-15 prescribes that at every section of a flexural member where tensile reinforcement is required by analysis, A_f provided should not be less than the area given by:

$$A_{fmin} = 0 \cdot 41\,\frac{\sqrt{f'_c}}{f_{fu}} b_w d > \frac{2.3}{f_{fu}} b_w d \;\rightarrow\; \rho_{min} = 0 \cdot 41\,\frac{\sqrt{f'_c}}{f_{fu}} > \frac{2.3}{f_{fu}} \quad (3)$$

Where b_w and d are the cross-section web width and the distance from the extreme compression fiber to the centroid of tension reinforcement, respectively.

It has been shown that the requirements of equation (3) may become unrealistic for large concrete cross sections. It is therefore suggested that the equation (3) need not be applied in a member where at every cross-section, the area of tensile reinforcement provided is at least one-third greater than that required by analysis.

4. Maximum FRP Reinforcement

Provision 10.3.5 in ACI 318-14 limits the minimum tensile strain at failure in the longitudinal steel reinforcement of flexural members to a value of 0.004, which corresponds to roughly twice the yield strain of a Grade 60 steel (420 MPa). This strain limit is to ensure that the failure of the steel-RC structural member will always be ductile. Even though this limit loses relevance in the case of FRP reinforcement, it may be argued that, irrespective of the fact that FRP bars do not yield, this strain threshold would at least ensure some visible level of distress in terms of deflection and crack width for a flexural member approaching failure. According to this provision, the maximum reinforcement ratio, ρ_{max}, for an FRP-RC member in flexure would be obtained as equation. (4)

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{0.004E_f} \frac{0.003}{0.003 + 0.004} = 91.1\beta_1 \frac{f'_c}{E_f} \quad (4)$$

5. Optimization Design

As it has been demonstrated experimentally, regardless of the reinforcing material used (FRP), the basic assumptions for the flexural theory of RC beams reinforced with FRP bars can be summarized as follows:

1. Plane sections remain plane; this means that shear deformations can be disregarded (Euler–Bernoulli beam theory)

2. A perfect bond exists between reinforcing bars and the surrounding concrete; in other words, the strain in the reinforcement is equal to the strain in the concrete at the same level.

3. Stresses in both concrete and reinforcement are presented based on the strain level reached in each material using the appropriate constitutive relations for concrete and reinforcing bars. In this study, the case of concrete, up to the serviceability limit state, a linear–elastic relationship will be used; past the linear elastic point up to crushing, either the Todeschini model or the equivalent stress block can be used. Regardless of the limit state considered FRP reinforcing bars is considered linear–elastic.

4. The tensile strength of the concrete is neglected.

5. The concrete is considered to fail when it reaches a maximum specified compressive strain.

The basic safety relationship at the ultimate limit state can be written as:

$$\emptyset M_n \geq Mu \ (5)$$

6. Optimization Method

The LMM method is employed to obtain the optimum design of the RC beam element. The problem's goal is to minimize the objective function presented in the following form;

$$s = f(x_1.x_2.x_3.\dots.x_n) \qquad (6)$$

Subjected Constraints:

$$h_i(x_1.x_2.x_3.\dots.x_n) = 0 \qquad i = 1.2.\dots.p \qquad (7)$$

Where n is the number of independent variables x_i and p is the number of constraints.

To solve the optimization problem based on equations (1) to (7), the unconstrained Lagrangian function L is presented in the following form:

$$L\left(x_1.x_2.x_3.\dots.x_n.\lambda_1.\lambda_2.\lambda_3.\dots.\lambda_p\right) = f(x_1.x_2.x_3.\dots.x_n) + \sum_{i=1}^{p} \lambda_i h_i\,(x_1.x_2.x_3.\dots.x_n) \quad (8)$$

where λ_p parameters are Lagrangian multipliers. The necessary conditions for Lagrangian function are as follows:

$$\frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} + \sum_{i=1}^{p} \lambda_i \frac{\partial h_i}{\partial x_k} = 0 \qquad k = 1.2.3.\dots.n \qquad (9)$$

$$\frac{\partial L}{\partial \lambda_i} = h_i = 0 \qquad i = 1.2.3. \ldots . p \qquad (10)$$

Expressions of (4) and (5) show that we have a system of n + p equalities with n + p unknowns. Therefore, their solution produces stationary values for x_1, x_2, x_3, …, x_n and λ_1, λ_2, λ_3, . . ., λ_p, in which an optimized solution would be achieved.

7. Applying the Lagrangian Multiplier Method

Reinforced concrete beams with FRP bars are primarily designed to bear the applied loading, which is mentioned in ACI 440-1R-2015 for singly reinforced concrete beams. In this study, minimizing the cost of the beam, which is subjected to a nominal bending moment Mn, is the objective, which means the cost of the beam is considered to be the objective function and the nominal flexural strength as the active constraint.

8. Singly Reinforced Concrete Beam

The cost function is formulated as an objective function to be minimized, after the nominal flexural strength of the RC beam is obtained and proposed as a constraint function. At last the Lagrangian function is presented and solved according to equations. (6) , (7), (8), respectively. Setting the ratio of the material costs to q= C_f/C_c, where Cf and Cc are the costs per unit volume, respectively. The total cost objective function per unit length, which is based on reinforcement area of the beam, geometry, and material costs, is as follows:

$$C = C_c b\left[\rho_{frp} q d + (1 + r)d\right] \qquad (11)$$

where ρ_{frp} is the reinforcement ratio As/bd, As is the area of tensile rebar, b and d are the breadth and effective depth of the section, respectively, and *r* is the ratio of reinforcement cover to effective depth *d*. b is assumed to be constant and concrete cover does not change in the design procedure. Based upon these simplification equations. (11) can be rewritten in the following form:

$$C = \rho_{frp} q d + (1 + r)d \qquad (12)$$

Flexural strength design at the section of a member, refereed as nominal flexural strength, of the member multiplied by the strength reduction factor (φ). Based on ACI 440-1R-2015 relation between nominal and ultimate flexural strength is:

$$\varphi M_n \geq M_u \qquad (13)$$

In the equation. (13), ϕ*Mn* is the factored bending moment capacity of the member and is based on the member's geometrical parameters, where the reinforcement is located, and the mechanical properties of the materials; the safety factors associated with the materials or the failure mode, depending upon the calculation procedures followed. The second term of Equation (13), *Mu,* is the factored bending moment resulting from the analysis of the member and is a function of the member. The nominal flexural strength at a section can be expressed in terms of FRP reinforcement ratio mentioned in ACI 440-1R-2015, as presented in the following equation:

$$\frac{M_n}{bd^2} = \rho_{frp} f_{frp} \left(1 - 0.59 \frac{f_{frp}\rho_{frp}}{f_c'}\right) \quad (14)$$

Where Mn is the nominal design moment, f_{frp} is the reduced strength of FRP bar , $\emptyset\ f_{frp}$, and f'_c is the reduced compressive strength of concrete, $\emptyset \mathrm{f_c}$, where $\emptyset_{frp}$, $\emptyset_c$, are the strength reduction factors of FRP and Concrete, respectively.

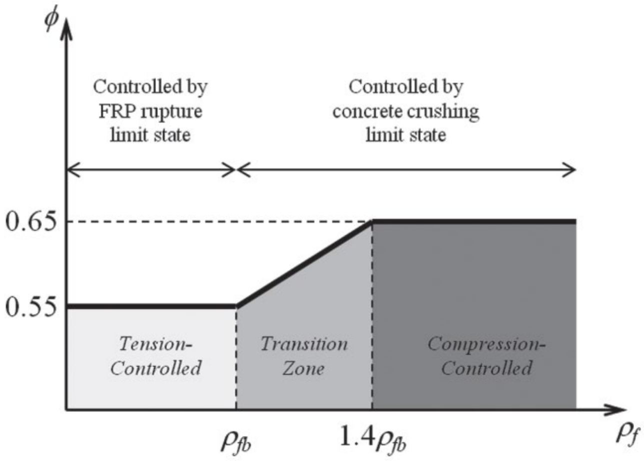


Figure 2. Strength reduction factor as a function of reinforcement ratio based on ACI 440.1 R 2015

The geometry of singly reinforced beam with stress block presented by (Todeschini, Bianchini, and Kesler) is shown in Figure 3.

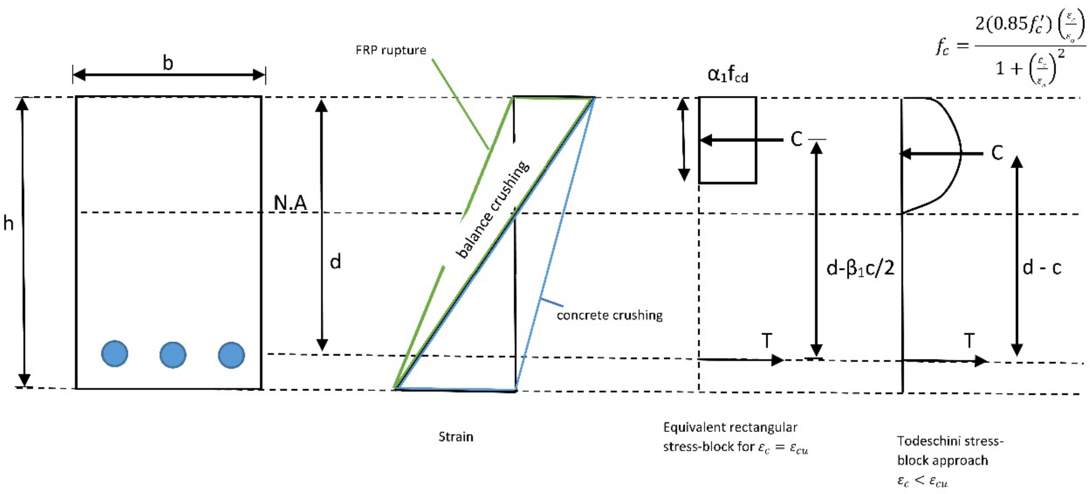


Figure 3. Geometry of a singly reinforced beam with stress block

The formulation of Lagrangian function and then solving the problem to achieve the minimized cost design is presented according to the proposed method, the unconstrained problem is formed using the Lagrangian function L according to the Eq. (3) as follows:

$$L = \rho_{frp} q d + (1+r)d + \lambda \left[\rho_{frp} f_{frp} \left(1 - 0.59 \frac{f_{frp}\rho_{frp}}{f_c'}\right) b d^2 - Mn\right] \tag{15}$$

The Eq. (10) is the Lagrangian function for singly reinforced concrete beam formed by applying partial derivatives of the Lagrangian function according to Eq.(10). By simplifying and equating the partial derivatives to zero. Then, substituting the equations, the optimum tensile reinforcement ratio of concrete beam is achieved as follows:

$$\rho_{frp\ opt} = \frac{50 f_c'(1+r)}{59 f_{frp}(1+r) + 50 f_c' q} = \frac{1}{1.18\left(\frac{f_{frp}}{f_c'}\right) + \left(\frac{q}{1+r}\right)} \tag{16}$$

Eq. (16) is used to obtain the optimum tensile reinforcement ratio for a singly reinforced concrete beam. Also, the optimum effective depth for a singly reinforced concrete beam is then derived by substituting equations (15) and (16):

$$d_{opt} = \sqrt{\frac{Mn}{b \rho_{frp\ opt}\ f_{frp}\left(1 - 0.59 \frac{\rho_{frp_{opt}}}{f_c'}\right)}} \tag{17}$$

Since Eqs. (16), and (17) are for singly reinforced concrete beam, it is essential to calculate upper bond $\rho_{\ frp_{\ opt}}$. According to ACI 318-14, tensile reinforcement ratio must be bonded by the allowable maximum reinforcement ratio presented in Eq. (16) which is the bond of limiting reinforcement ratio for which both FRP and concrete gain their ultimate strength assuming Euler-Bernoulli's principle and linear behavior. The compressive reinforcement ratio must be bounded by the allowable minimum reinforcement ratio as in ACI regulations.

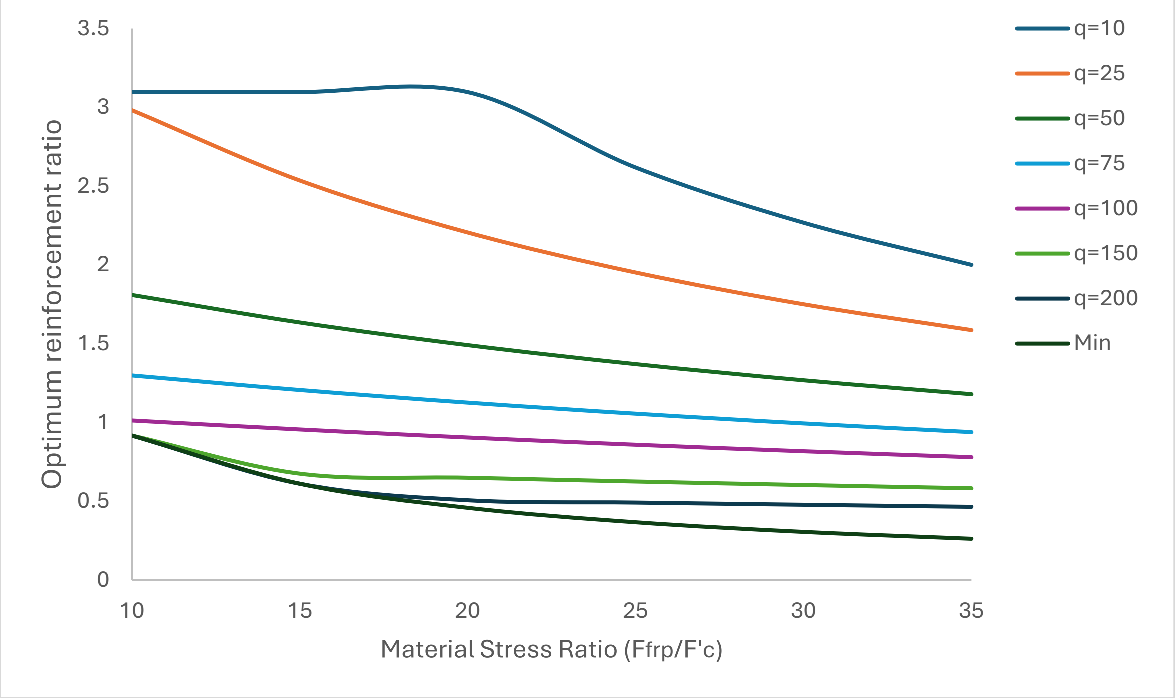
$$\rho_{\ frpb} = 0.85\,\beta_1 \frac{f_c'}{f_{frp}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{frp}} \quad (18)$$

$$\rho_{\ frpb} = 0.85\,\beta_1 \frac{f_c'}{0.004 E_{frp}} \frac{0.003}{0.003 + 0.004} = 91.1\,\beta_1 \frac{f_c'}{f_{frp}\,E_{frp}} \quad (19)$$

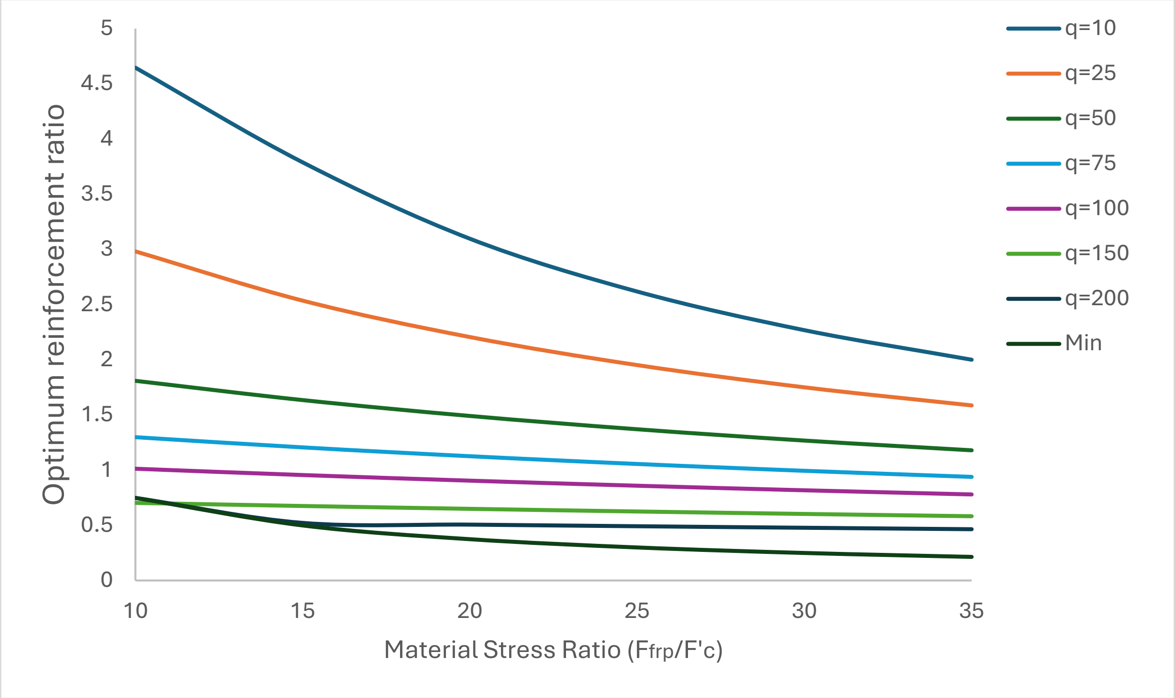
It should not be neglected that the value of maximum reinforcement ratio cannot be practically applied for GFRP reinforced concrete flexural members since it is impossible to fit may bars in the cross section.

The represented optimum tensile reinforcement ratio for a singly reinforced concrete beam is given by Equation (16). A group of lines has been drawn for various material cost ratios q for a constant value of r = 0.15. The graphs are restricted by the maximum and minimum limitations of $\rho_{\ max}$ and $\rho_{\ min}$ on the reinforcement ratios given by ACI 318-14 and ACI 440-1R-15. It must be noted that these limitations are the boundary constraints in the process of optimization analyses, and the nominal flexural strength is the main constraint. Thus, in the process, it is considered to be active.

In Figure 4, optimal curves for r ranged from 0.05 to 0.2. Figure 4 shows compression values ranging from 35 to 50 MPa, q = 25, and r = 0.15, covering the full ACI range. For other values of q, it is possible to mathematically derive the valid stress ratio range for each design type. For example, Table 1 is derived using values of 0.15 for r.



a)$f_c' = 20\ MPa$



b)$f_c' = 30\ MPa$

Figure 4. Optimum tensile reinforcement ratio for singly reinforced concrete beams

9. Sensitivity Analysis

In this section, the optimum solutions for singly reinforced concrete beam sections for different values of material stresses are compared, and various practical design solutions are presented.

The reinforcement ratio $\rho_{frp_{opt}}$ is restricted by the bonding value $\rho_{frp_{balanced}}$. Through which the following inequality expression is obtained:

$$91.1\,\beta_1 \frac{f_c'}{f_{frp}\,E_{frp}} \leq \frac{50f_c'(1+r)}{59f_{frp}(1+r)+50f_c'q} \qquad (15)$$

Based on Eq. (15)

Material Cost Ratio	Single Reinforcement Optimum Range	Boundary Reinforcement Optimum Range
(q)	f_f/f_c	f_f/f_c
25	5.0-9.2	9.2-13.4
35	5.0-12.8	12.8-18.8
45	5.0-16.5	16.5-24.1
55	5.0-20.2	20.2-25.0
65	5.0-23.8	23.8-25.0
75	5.0-25.0	Outside the practical range (>25)
85	5.0-25.0	
95	5.0-25.0	

Design Example 1 - Singly Reinforced FRP Beam A beam of width b=260 mm is subjected to the maximum bending moment of 185 kNm. The ratio r is taken as 0.15, the material cost ratio q as 75, and the costs of concrete Cc as 50 \$/m^3. Characteristic strength of FRP bar and concrete are 460 and 30 N/mm^2 respectively, giving a material stress ratio f_{frp} /f'_c of 15. The lower- (d_l) and upper- bound (d_u) effective depths are taken to be 300 mm and 800 mm, respectively.

Using Figure 6, the optimum solution is shown to be a singly reinforced section. Hence, from Eq. (11) ρ_{opt} is 0.01206, giving the corresponding optimum effective depth of the section d_{opt} obtained from (12) as 379 mm. The required area of the reinforcement A_{frp} req is calculated to be 1189 mm^2. The corresponding total material cost of beam per unit length C is then obtained from (6) to be 0.203 Cc \$/m at its minimum.

A graphical representation of the results is given in Figure 5, showing the optimum to lie on the bending stress constraint boundary with the cost objective function.

The feasible answer is bounded by the bending stress constraint, the upper bound effective depth, and the maximum area of reinforcement, $A_{frp\ max}$, which corresponds to the intersection of the boundary reinforcement line with the bending stress constraint. Table 2 shows the results using the standard design approach. It is evident from this table that the derived optimum design formulae for singly reinforced sections give an accurate estimate of the minimum material cost.

Design Number	Effective Depth d (mm)	Area of Tension Reinforcement A_{frp} (mm²)	Total Material Costs (C_c) ($/m)
1	300	1649.17	0.2134
2	320	1494.32	0.2078
3	340	1371.06	0.2045
4	359	1274.20	0.2029
5	380	1183.90	0.2024
6	400	1110.35	0.2029
7	440	989.83	0.2058
8	540	783.46	0.2202
9	580	724.18	0.2277
10	640	650.93	0.2402
11	680	610.06	0.2491
12	760	542.35	0.2679
13	800	513.96	0.2777

Table 2. Comparison between the LMM and standard design approach – Design Example

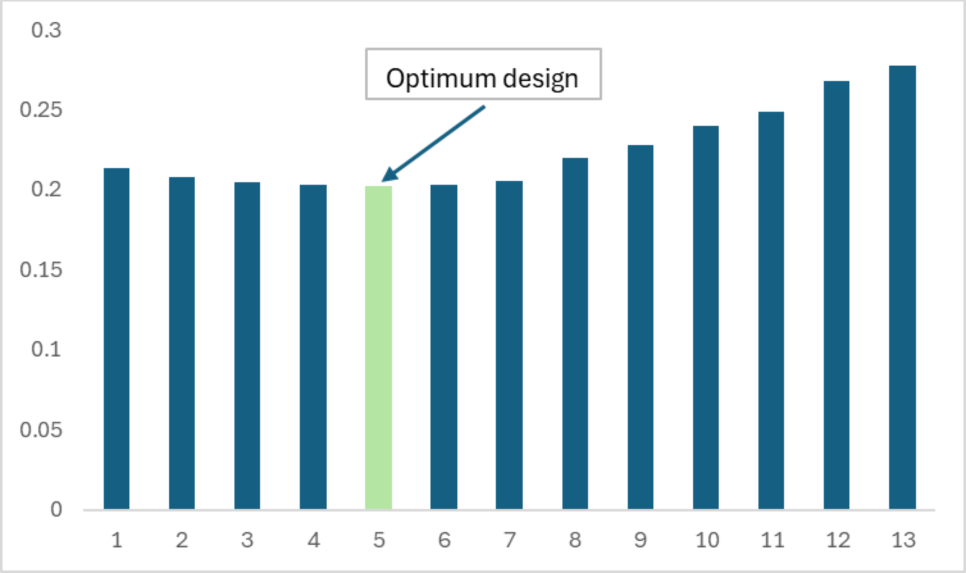


Figure 5. Comparison between the optimum and conventional designs for the design example

10. Conclusion

In this study, the implementation of minimizing the cost design of reinforced concrete beam elements using FRP bars is dealt with. The objective function was set on the total cost of concrete and FRP materials. The beam's resistance against the nominal bending moment was considered as the constraint for the optimization process. The minimum cost design was achieved using the Lagrangian Multiplier Method (LLM). The results were presented in closed-form equations that can be easily used by scientists and engineers. The application of the proposed relations and curves is demonstrated through one real-life example of an SRB design situation, and it is shown that the minimum cost design is actually reached using the proposed method. Meanwhile, the results were presented in graphical form. presented relations derived to easily find the optimum design solution. Furthermore, sensitivity analysis of the total cost revealed that the total cost varies considering the FRP to concrete cost ratio for a singly reinforced concrete beam. Finally, to illustrate the beneficial aspects of the proposed method, a real-life example of reinforced concrete beam design was presented. It was shown that a designer can easily achieve an optimum design by choosing the material cost and stress ratios using closed-form equations or presented design graphs.

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