

Measurement of Cable Stays Tension Forces by Ambient Vibration Testing and Lift-Off Method

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Abstract

Cable-stayed bridges have become one of the important parts of today's construction style as they are preferred over their aesthetics as well as their way of construction. Over longer distances, cable-stayed bridges have proven to be one of the best types to withstand any kind of loading. Cable-stay bridges have gained popularity nowadays, but the construction and maintenance of these bridges require extra effort. For this reason, monitoring bridges is a must for the betterment and safety of the public. A usual method that is being used by the engineers is the 'vibration method, which measures tension during the construction of the cable system stiffened with inclined cables. A basic formula is applied here, which provides the tension force among the cables, depending upon the cable sag and vertical angle effects. This project puts forward the analysis of cables that measure the tension of steel in a cable-stayed bridge using appropriate methods, such as the Ambient Vibration Test, which provides the best possible outcome. The analysis of the bridge was done on a cable-stayed bridge, which was constructed in 2008 with an overall span length of 1708m. The results obtained from the analysis done by testing companies A and B were compared with the as-built results, which were obtained by the Lift-off Method, which showed an error of 0.18 and 0.23, respectively, which are within the range of the Lift-Off data set. This study concludes that Ambient Vibration Testing using the Taut String Formula can be used to calculate cable forces due to its higher precision with Lift-Off data.

Keywords: Cable-Stayed Bridge, Ambient Vibration Testing, Lift-Off Test, Taut String Formula

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1. Introduction

Among the distinct forms of prestressed concrete bridges available, one of the most varied forms of bridges is the cable-stayed, which now no longer handiest offers precise structural association however actually have a higher aesthetic view. For a cable-stayed bridge, the cables act as the principal source of load-bearing elements. Thus, cable integrity is related to the safety of a bridge, which may be accessed via means of calculating the tension force. During the service life, the cable would possibly be subject to corrosion and weariness thereby degenerating the cable, leading to dwindled resistance (i.e., tension) by the cable. This necessitates the regular measurement, frequent checking of cable tension for the safety and sustainable operation of a cable-stayed bridge (Oh et al., 2024). There are methods of calculating the cable tension, both directly using load-measurement devices (known as the Direct Method) or determined from responses of the cable (known as the Indirect Method). In the case of the Direct Method, the cable tension is measured by the use of a load cell or a pressure meter connected to the cable at the onset of a cable. The Direct method may be used in cables with a sag or bending stiffness, or in the ones with no sag and flexural rigidity. (Zui et al., 1996; Russel and Lardner, 1998).

Studies from the past reveal that many efforts have been made since the 1950s to ascertain cable forces established through a simple form of vibration, and using Taut String Theory, a fundamental relationship is established between tensile force in cables and the induced frequency of the cable (Saxon & Cahn 1953; Irvine 1981). According to (Kangas et al., 2010; Cunha et al., 2001; Mehrabi, 2006) ambient vibration test is among the constructive methods to observe the functioning of structures as a non-destructive test, as well as its benefits of cost minimization and directness. According to (Cho, Yim, et al., 2013), the Lift-Off Test is costly and potentially damaging to assess these forces by static means using Lift-Off testing by hydraulic jacks; this technology is therefore not very widely used in civil engineering practice and alternatives are needed.

This paper covers the approach of getting the best possible way of finding cable force that can be easy, cheap, and accurate. The performed test results were then compared with each other to find the accuracy of the test. Further on, an error analysis was done in Ambient Vibration Testing (AVT) with a Lift-off test to choose an appropriate approach towards the calculation of cable forces using the Taut-string Formula.

2. Literature Review

2.1. Ambient Vibration Testing (AVT)

According to (Kangas et al., 2010; Cunha et al., 2001; Mehrabi, 2006) ambient vibration test is among the constructive methods to observe the functioning of structures as a non-destructive test, as well as its benefits of cost minimization and directness. The measurement of vibration

of the structures done with the help of vibration test is mainly the approximation of structural modal parameters which are mainly shape, oscillation, and damping ratio. The test outcomes mostly rely on how the tested bridge is excited and vibrated to gauge the bridge retort for such excitation and further process the derived data to acquire the dynamic properties.

The use of Ambient Vibration Test is seen in many past studies since it is faster, simpler, consumes less money, and does not disturb the traffic while conducting the test (Afzal & Javed, 2024). Moreover, its non-destructive requires less labor and is conducted with ease. AVT has been conducted on various structures and one of the key components used in this testing is an accelerometer which is used to capture the vibration generated within the cables. The existing vibration methods for cable tension measurement may be classified into forced vibration and ambient vibration. By getting all the desired frequencies from the cable, the Taut String Theory can be utilized by using the Bi-square Linear Fit Method to calculate the cable forces (Jiang, Kim & Ono, 2023).

$$T = 4 \times m \times L^2 \times \left(\frac{f_n}{n}\right)^2$$

in which f_n = nth natural frequency in Hz; and T, m, land n refers to a tension force, mass density, length, and mode number of cables, respectively. This idealization undoubtedly simplifies the analysis but may introduce unacceptable errors by ignoring the sag and bending stiffness of the cables (Casas 1994).

2.2. Lift-Off Test

The Lift-off Test is a mechanical process that uses a small-scale load cell, a hydraulic jack, and a displacement meter to determine the force in a cable (for a single wire or a strand) (Sun, Chen & Huang, 2022). After it has been stretched, a lift-off is used to check the strength of a tendon or cable. A lift-off test may be needed if the elongation of recently stressed tendons or cables is beyond the code-recommended tolerance of +/- 7 percent. Before the stressing tails of the tendons are cut off and in development, the lift-off approach can be used.

Usually, the Lift-off Test is used to calculate the tension of a small number of strands in the cable. In the stay cable, the tension of each strand is then assumed to be the mean value of measured tensions. Multiplying the acquired mean value for strand tension by the number of strands inside the stay cable, the total stay cable tension is determined. Using "lock-off" wedges, strand ends lock at anchor holes due to the tension force within each strand.

The Lift-Off test data is used then to calculate cable forces using the equation below, cable forces can be calculated.

$$T = \frac{\sum_{i=1}^{n_s} T_i}{n_s} \times n$$

Here T is the tension of the cable, which is to be calculated, T_i is the Lift-Off test tension of the strand which is generated using the hydraulic jack and recorded by load-displacement meter, n_s is the number of strands that have undergone the Lift-off test and n is the total number of strands inside the cable.

3. Methodology

AVT has proven to be the best option for testing any structure due to its cheap, and accuracy. The use of ambient vibration test is seen in many past studies since it is faster, simpler, consumes less money, and does not disturb the traffic while conducting the test. Moreover, its non-destructive requires less labor and is conducted with ease. The Lift-Off test is considered to be expensive and takes more time to perform, but the results obtained by this are way closer to the design-built data.

3.1. Bridge Description

The cable-stayed bridge is a semi-fan style bridge constructed with a composite deck in the middle and a concrete deck on the ends. The cables provide the stability to the bridge to withstand any load imposed on the bridge. The concrete deck is a semi-box girder with a twin plane of stays. The pylon height of the bridge goes to 143m and the overall length of the bridge is 1708m. The main span of the bridge, which is the composite deck is across the river with a length of 500m. The shape of the pylons is 'A' shaped, which has an impact on the cable arrangement as well as aesthetics of the bridge. The bridge itself is a semi-fan style bridge constructed with a composite deck in the middle and a concrete deck on the ends. The bridge has 42 cables on the east side and 42 cables on the west side. Figure 1 shows the cable that was tested in the study.

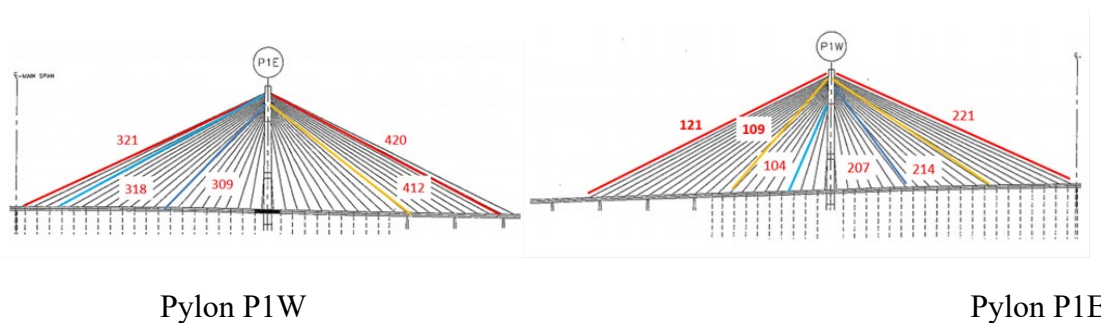


Figure 1. Selected stay cables tested

3.2. Ambient Vibration Testing Procedure

For testing, the two most important things are needed to continue the procedure of AVT, a wireless sensing unit which is known as an accelerometer (uniaxial or triaxial), and a computer with vibra-test software which caters to the acceleration and converts it to frequency. The sensing unit is placed at a certain distance on the cable, mainly 3-5 m above the deck level using a cherry picker to reach this height. The sensor is attached to the cable which further is connected to the vibra-test box, as a transducer that collects the data and sends it to the computer. Once the sensor is set and placed tightly around the cable, the testing starts and the computer starts to record the acceleration vs time domain, which alternatively changes to frequency vs time domain. Testing is done up to mode 6 which provides a good opportunity for results to be achieved. Once the testing starts, the data acquisition is performed for approximately up to 90 sec for each mode. As soon as testing up to the desired mode is done, data acquisition is stopped, and using inbuilt tension force estimation in the software, the tension force is calculated using the Taut String theory principle.



Figure 2. Accelerometer placement on cable.

3.3. Lift-Off Test Procedure

The lift-off test is performed by setting up the hydraulic jack to the extra tendon length of the anchor head and applying load. First, the extra pooling head is set, this creates a function for fixing the extra tendon length of the anchor bar and the tension bar. The Lift-off Test then is performed by applying load on the tension bar.

The hydraulic jack initially pulls the free end of the strand with an increasing force, and the corresponding strand displacement is simultaneously reported (the force-displacement curve is plotted). Until the applied force of the hydraulic jack exceeds the strand tension force, the strand end stays in its position. The lock-off wedge pops out of the anchor block hole at this stage. "This condition is referred to as "lift-off". The slope of the force-displacement curve will

decrease until lift-off occurs. Consequently, the force at the transition point of the slope is known as the stress of the strand.

In this process of loading, once the residual tensile force is checked, the process of unloading is performed. Once the loading and unloading are performed, the relationship between the force applied and displacement is measured by the displacement meter. As shown in Figure 3 firstly, before the anchor head starts to lift away, the graph is the linear line from point O-A, and then as soon as the load on the hydraulic jack increases, becomes equal to the residual force, this is the point after A where the anchor head starts to lift away from the bearing plate. The value which is measured in the load-displacement curve is known as the lift-off value.

Once the lift-off happens, the linear slope which was obtained from O-A gradually changes and goes towards A-B, as soon as this change becomes constant, the load gets concerted to anchor free length. At this time, the load-displacement goes again to the linear slope gradually which depends upon the elastic modulus of the anchor.

After the process of loading is completed, which is achieved once the residual tensile load of the strand is calculated and confirmed, unloading is performed which can be seen in the graph from D-E-F.

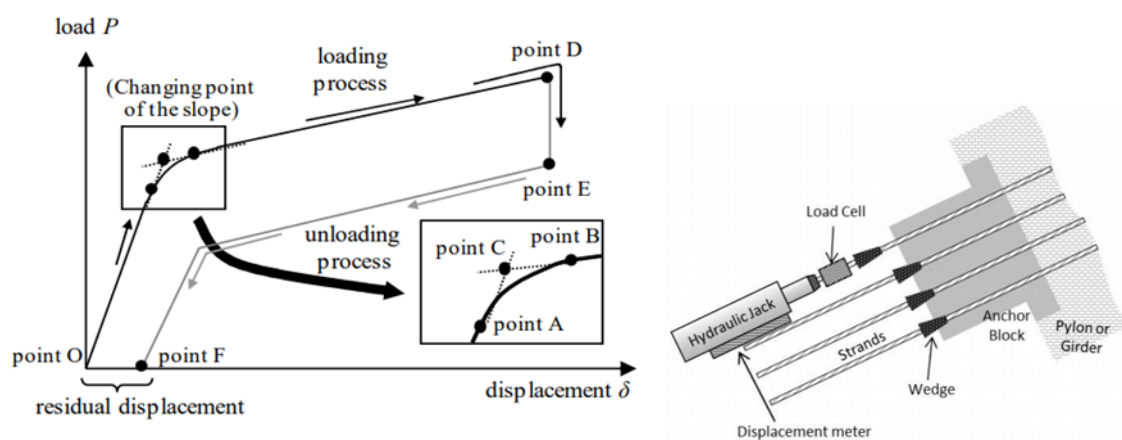


Figure 3. Load-Displacement curve in Lift-Off test and placement of Hydraulic jack.

4. Results and Analysis

4.1. Ambient Vibration Test Results

By vibration measurement, using a sensor fixed near to the cable anchor and therefore close to the vibration node it is possible to identify natural frequencies and damping coefficients using extremely sensible equipment. And in this area only minor vibration amplitudes exist, the position is beneficial for technological and economic purposes (accessibility, installation time, etc.). To obtain high-quality data with enough signal-to-sound ratio, it is important to use very

sensitive accelerometers and secure cables. A rope's vibration reaction either occurs in the cable plane or out of it. The frequencies obtained for both directions are slightly different as boundary conditions may vary (anchorage, free vibration length) and cable weight (cable sag) only influences in-plane modes. These effects must be considered for analysis and cable force measurement.

For the AVT, two companies (company A & B) did the testing using the same procedure and formula for finding the cable forces.

For company A, natural frequencies are defined using ARTeMIS, commercially available software for Operational Modal Analysis and Experimental Modal Analysis, from ambient vibration response measurements of the cables. In ARTeMIS, acceleration time history data undergo multiple signal processing steps during the identification process using the Enhanced Frequency Domain Decomposition (EFDD) technique, such as spectral density estimations and correlation function matrices, followed by singular value decomposition (SVD) of the spectral density matrix. Then, the peaks of the singular values that represent the vibration mode are selected in the plot. The curve-fits of the selected peaks are then immediately calculated by EFDD, which will yield the natural frequency of the corresponding vibration mode.

In Russell and Lardner, experimental validation of the determination of tension force via natural frequencies was first published. The cable tension was measured based on the determined natural frequencies using the Taut String Theory:

$$T = 4 \times m \times L^2 \times \left(\frac{f_n}{n}\right)^2$$

Where:

T = tension force in the cable.

m = mass per unit length of the cable.

f_n = natural frequency of vibration at the n th mode.

L = effective cable length.

n = mode number associated with the specified natural frequency.

Note that in the cable tension calculation for this frequency-based process, the fundamental frequency ($n = 1$) is used. In general, the error of T is greater as the order of frequency is higher. However, when using different order frequencies, the T errors are distinct/vary (within 5 percent of errors).

Table 1 shows the average natural frequencies from the two data sets and used for the cable tension calculations.

Table 1. Average of Data set 1 and 2 for Natural Frequency.

Cable No	Free Length, L (m)	Mass per Length, m (kg/m)	Natural Frequency Measurement Data Set 2, f_n (Hz)				
			Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
104	97.376	54.6	1.294	2.560	3.863	5.131	6.411
121	268.259	98.4	0.495	0.990	1.489	1.979	2.473
207	117.930	59.8	1.039	2.077	3.108	4.146	5.186
214	185.272	89.3	0.680	1.355	2.031	2.701	3.376
221	262.376	79.0	0.489	0.987	1.484	1.976	2.466
309	135.521	67.6	0.901	1.798	2.690	3.577	4.485
318	228.497	98.4	0.586	1.133	1.710	2.287	2.842
321	262.376	79.0	0.509	1.015	1.522	2.027	2.539
412	169.284	80.3	0.732	1.428	2.161	2.881	3.613
420	256.628	97.1	0.513	1.026	1.540	2.054	2.568

Table 2. Cable Force Data

Cable No	Effective Length, L (m)	Tension Force, (kN)	Cable No	Effective Length, L (m)	Tension Force, (kN)
104	97.38	3468	319	239.63	6990
121	268.26	6926	307	117.93	3338
207	117.93	3591	407	121.45	4176
214	185.27	5661	421	267.36	6935
221	262.38	5202	121	269.16	7068
309	135.52	4027	109	139.57	4199
318	228.50	7045	103	89.52	3374
321	262.38	5625			
412	169.28	4932			
420	256.63	6732			

Company A used the Vibra Box mounted on the cable to measure the frequency of the cable. This natural frequency was then used to find the cable forces using the famous formula proposed known to be Taut String formula, six modes of frequencies were recorded (6-inplane, 6-out plane), and then using these frequencies forces of cable were found and an average value of calculated forces was taken up for the comparison. The calculated cable forces for each cable are shown in Table 2.

The same concept was also used up by the Company B in the calculation of the cable forces and for performing the data collection. The frequency measurement was done through the Vibratest software which plots the acceleration vs frequency graph and later using those frequencies, the cable forces are calculated. Taking note here the final result of force is

calculating the forces for each mode and taking an average value. The accuracy of the result depends here upon the three parameters mass, length of cable found, and finally frequency.

The cable force measurement done using the taut-string formula does not consider the use of sag or stiffness parameters which can affect cable force but still taut-string can be found to be giving promising results.

4.2. Lift-Off Test Results

The lift-off test was performed back in 2011 with the view to check whether the values found through design and actual were similar. On this view when the testing was done it was found that the values are quite similar to that of design-built values. This is because the Lift-off test is done by loading and unloading force subjected to the strand of the cable itself.

When the testing was done, it can be found that since all the strands have the same properties, thus, can be subjected to similar tension if applied. Thus, this concept provides the best way by applying the loading and unloading process to a selected number of strands which later can help in the calculation of cable force. Considering that there are 71 strands in cable number 121, 5 out of 71 strands are taken to perform the Lift-Off test, and then using the equation below, cable forces can be calculated.

$$T = \frac{\sum_{i=1}^{n_s} T_i}{n_s} \times n$$

Here T is the tension of the cable, which is to be calculated, T_i is the Lift-Off test tension of the strand which is generated using the hydraulic jack and recorded by load-displacement meter, n_s is the number of strands that have undergone the Lift-off test and n is the total number of strands inside the cable.

4.3. Comparison of AV And Lift-Off Data

Comparing the company A and B and Lift-off data it can be seen that the results are in range. From Fig. 4 which shows the graph of AVT and VSL data, it can be seen that the range set up for the comparison is shown by the purple line which is the lift-off cable forces also known as as-built data. This data set is taken up as a range for comparison as Lift-Off deduced cable forces are quite similar to design-built cable forces of the bridge. The range taken can be seen by plotting the values of as-built data which will act as maximum values for each cable, as they are precisely similar to the design-built range, and that will be the range for comparison of data of AVT. Company A & B provided outstanding values which can be used to suggest the AVT as an easy maintenance procedure instead of the Lift-Off test. For the comparison of values,

two cables were taken up, 121 and 109, and the cable forces were found out using taut string formula by both company A & B, it was found that the results by both of the companies are in an acceptable range and as such it can be said the bridge is still in better condition as it was supposed to be. Here also talking on how the forces were calculated by each of the company, Company A used the average values of frequencies that is they performed the testing twice and took the average for frequency for each mode and then calculated the cable force, on the other hand, Company B, they performed the test only once and the found out force for each mode and took the average value of those calculated forces to generalize the cable force as such it can be seen that there is a slightly higher possibility of error.

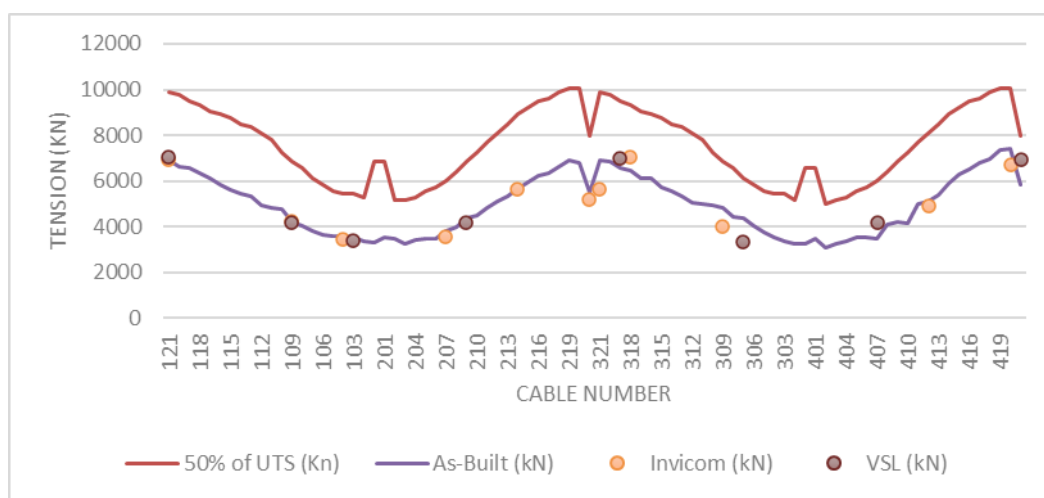


Figure 4. Graph Lift-Off data, Company A (Invicom) forces, and Company B (VSL) forces.

4.4. Error Analysis

Estimated natural frequencies showing very near precision for both datasets suggest that the measurement is accurate and reliable. In the estimation of cable tension, the use of average natural frequencies increases the statistical reliability of the performance. There is a very similar and reliable relation between the cable voltage measured from the frequency-based calculation and the total cable voltage using the lift-off test results. As seen by many other publications, the percentage of errors in the estimation of cable tension using frequency-based measurement is within a reasonable limit that is in the range of 0.4-5.6 %.

The accuracy of cable force calculation depends basically on 3 main parameters and accuracy can be obtained by taking the partial derivative of the Taut String Formula,

$$\frac{\Delta T}{T} = \frac{\Delta m}{m} + 2 \frac{\Delta L}{L} + 2 \frac{\Delta f}{f}$$

Where the following implies:

$\Delta T/T$ the relative error from measuring cable tension.

$\Delta m/m$ the relative error from determining the distributed cable mass.

$\Delta L/L$ the relative error from determining the length between two fixity nodes.

$\Delta f/f$ the relative error from measuring the frequency.

From the above-mentioned derivative, it can be observed that the accuracy depends on cable mass, length, and frequency. The accuracy of mass can be calculated at 2%, cable length measured with a precision of 0.3%, and the frequency measured with a precision of 1/200 (easily achievable, the frequency precision depends on the synchronization precision of data logger, which is near to be perfect, especially if the Fourier is performed on a higher number of points), the relative error made on the tension can therefore be in a range of $2.0\% + 2*0.3\% + 2*.5\% = 3.6\%$ (VSL, 2019).

Company A was originally assigned to conduct a 10-stay cable evaluation. As it was appropriate to verify the findings with a higher number of wires, additional cables were weighed. Consequently, 2 cables (cable 121 and cable 109) were reference cables to be checked by the cable manufacturer's measurements, which are compared and shown in Table 3.

Table 3. Error between Lift-Off vs Company A and Company B vs Company A

Cable No	Lift-Off Force (kN)	Company B Force (kN)	Company A Force (kN)	% difference between As-Built & A	% difference between Company B & Company A
121	6887	7068	6926	0.5%	2%
109	4233	4199	4229	0.09%	0.71%

The calculation found that the two percentage differences between the measurements of As-Built Vs Company A and Company B Vs Company A are very small and within a reasonable range.

The use of the ARTeMIS software package helped to imagine the natural frequency shape, although using a single reference point. The shape helped to decide whether the mode is the transverse or in-plane mode. To avoid inaccuracies resulting from geometry, movement restrictors, and gravity, only one type of mode should be taken as a guide to decide the tension of the cable.

Although the Fourier transform has calculated many natural frequency modes, only one frequency is used to measure the voltage instead of measuring the voltage from many frequencies and averaging to obtain the average voltage value.

Figure 5 shows the absolute error calculated for both Company B and A showed a broader picture of the discussion of whether to use force average value or frequency average value. Although the acceptable absolute error can go up to 3.6 % with a slight difference if occurred in the calculation of various parameters like mass, length, and frequency. It can be seen that the highest error was found for the Company B data which indicates that the average value of forces to be taken can provide more errored data which can allow wrong decisions for various things. It was found that the highest absolute error for Company B was 0.23 and that of Company A was 0.18 this indicates that values are in the acceptable range but Company A way of calculating cable force is considered to be a better way. Take note here that the Taut String formula that is being used for the calculation of cable forces doesn't consider any sag of stiffness effect which can also cause the deviation in the values.

An overall limitation that can be said which was found here was that the modified Taut String formula should have been used to achieve more accurate results. These errors are still considered to be in an acceptable range and as such further decisions can be made with full confidence.

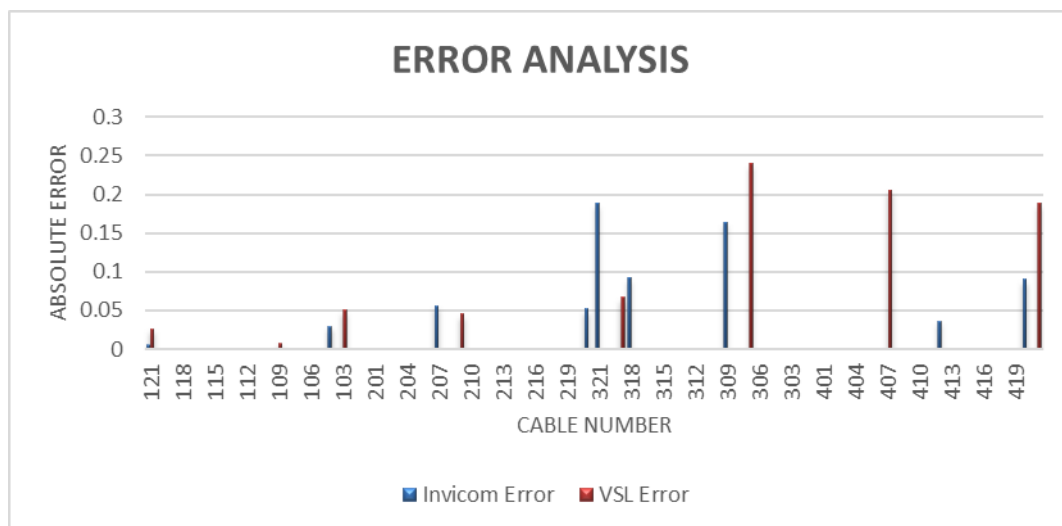


Figure 5. Maximum and minimum absolute error.

Many frequencies indicate varieties of damping and complexities and non-ideal string geometry estimation (De Iuliis et al., 2023). Only the simplest mode with the accurate frequency with low complexity was taken to calculate the tension and reported. Averaging the results from many frequencies will only introduce greater error in the calculation.

In the estimation, only the same modal frequencies were averaged between the various data sets, suggesting an incredibly low standard deviation for the average measured data.

5. Conclusion

In this study, a comparative study and analysis were done by performing two tension measuring tests which were carried on a real cable-stayed bridge. Using load cell Lift-Off test was performed and using an accelerometer ambient vibration testing was done. All the tests were performed on a cable-stay bridge with composite and non-composite deck, and with semi fan cable arrangement.

The results from this study which was performed by the two companies showed a satisfactory result which can help deduce the cable forces. A simple formula- taut string theory can be used to find the cable forces once the frequency, cable length, and mass per length are determined. When the tests were compared with the Lift-Off result it can be seen that the error between the AVT data and Lift-Off data was not that much with an absolute error of 0.23 for (Company B) vs Lift-Off and 0.18 for Company A vs Lift-Off. This comparison was done through absolute error calculation between the two data sets and the Lift-Off data set. This implies that both testing procedures can be put forth for calculating the cable force. The relative error for the AVT can be up to the range of 3.6% (if the difference between the parameters is changing). The AVT has shown a significant result and thus can be used to perform the tension testing of the cables. AVT is proven out to be cheaper and less labor and cost-effective method which can provide or can estimate the tension quite similar to design tension.

It can be concluded that to ensure that bridge performance can be measured in current and future changing conditions and that the assessed outcomes can facilitate the planning of bridge-inspection activities, at least a structural health monitoring system for a long-term bridge can be able to track the loading- and structural requirements defined by the bridge engineer.

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