May 2025
Volume 1
Issue 1

Interdisciplinary Journal of Civil Engineering
Shahid Beheshti University

DOI: 10.48308/ijce.2025.105775

Design and Analysis of Riverbank Filtration Systems Using Linear Systems Response Functions

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Abstract

Riverbank filtration systems offer a useful and reliable method for meeting domestic and industrial demands. In these systems, some wells are constructed in the bank of a river, where the water that flows across porous media into them has a very low pollutant level compared to the river. This study develops a cost-effective optimization model to minimize total cost. A simulation model for analyzing these systems is developed as well. In these models, the analytic solutions of the groundwater flow equations and pollutant transport are used. Using the concept of response functions of linear systems, these solutions are generalized in the case of variable pumping. In common RBF systems, the unit pulse response function of drawdown is independent of wells' location, and the transient flow equation reaches pseudo steady-state conditions. Two hypothetical example problems are presented; in the first, the design of a system is considered for meeting a given demand. The model solutions give the distance of wells' alignment from the river, the distance between wells, and the wells' pumping rates. The model also outputs the pollutant concentration in the wells. The results of the steady optimization problem reveal that, unexpectedly, the central well's discharge is greater than the side wells' discharges. The resulting pumping, conveyance, and treatment costs showed that all three cost terms are important. The sensitivity analysis revealed that all four considered parameters are sensitive, with the sensitivity ranking of: T (transmissivity), λ (decay rate), Θ (porosity), and R_d (retardation factor). In the second problem, an existing RBF system was analyzed by a simulation model, and the variations of the well's concentration were assessed by altering the four sensitive parameters. The proposed models are useful tools for primary design and analysis of RBF systems and assessing the effects of changing parameters on the system behavior.

Keywords: Riverbank Filtration, Response Function of Linear Systems, Analytic Solution, Optimization

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1. Introduction

Rivers are one of the most important sources of domestic water supplies. Entry of different pollutants, such as agricultural, industrial, and urban wastewaters, into the rivers dramatically compromises their quality. Typically, a great deal of budget is spent on the treatment of river water withdrawals in water treatment plants.

Riverbank filtration (RBF) is one of the efficient methods for the pretreatment of river water pollution. RBF systems are widely used for drinking water provision and treatment in several cities around the world, especially in European countries, where they provide a cost-effective and sustainable alternative compared to direct surface water intake and treatment. As illustrated in Fig. 1, in this method, some wells are constructed adjacent to a permanent river, which together have the capacity to meet a given demand. During transport and seepage of water from rivers to wells through porous media of riverbanks, a considerable level of pollutants can be removed. Although this method is largely classified as physical treatment, in most cases, it removes or reduces most chemical and biological pollutants.

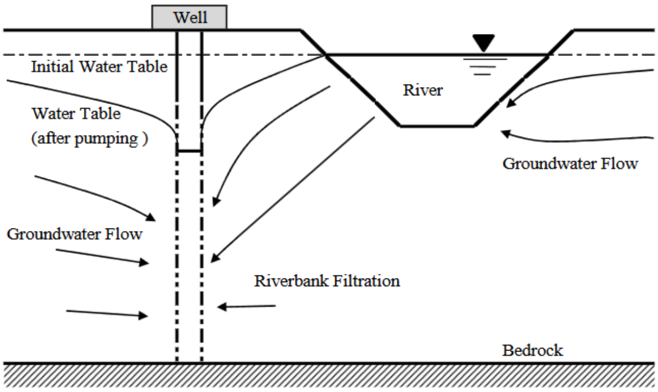

Figure 1. Schematic of an RBF system

Another important feature of RBF systems is their use in accidents through which dangerous pollutants, such as chemical tanks, enter the river. These events have less intense effects on the performance of RBF systems because of the relatively long delay for river water reaching wells. In conventional RBF systems, vertical wells are usually constructed in a straight line parallel to the river line; however, in some situations, the use of horizontal (collector) wells increases the discharge rate to wells (Ray et al., 2003).

Most RBF systems are constructed in alluvial sandy aquifers (banks). In addition to enhancing water quality, reducing water temperature, and protecting fish and other aquatic creatures are other advantages of RBF systems. The major elements of an RBF system that must be considered include the number of production wells and their capacities (discharges), the distance between the river course and the wells' alignment, and the distance between wells.

When the desired quantity (discharge) and quality (concentration) are given, the system could be designed with a minimum cost (cost-effective design). As demonstrated in Fig. 2, the system costs include the costs of constructing wells and OMR, the costs of constructing pipelines and OMR, the pumping cost of wells, and the water treatment cost.

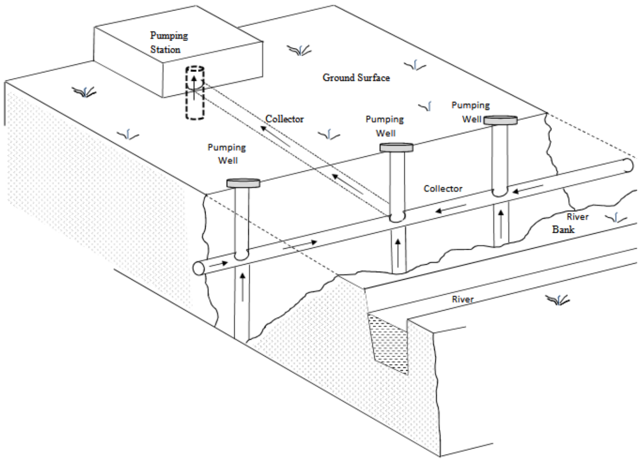

Figure 2. Component of an RBF system

2. Literature Review

It appears that the Glasgow Water Supply Company was the first known firm to develop and utilize an RBF system in 1810 (Ray et al., 1999). Moser et al. (1990) studied an aquifer that used two adjacent rivers for an RBF system. Using an advection-dispersion-based model, they computed the transfer time from the river to the aquifer, and they concluded that by knowing this delay time, there is enough time to control and prevent river pollution of the pumping wells.

Doussan et al. (1997) assessed the general characteristics of RBF systems in a portion of the Seine River in France. They simulated the oxidation and reduction reactions as well as the nitrogen transfer reaction, using a numerical model. They reported the importance of the rate of water (discharge), sediment, and organic carbon in the quality of transferred water.
Dillon et al. (2001) studied the potential of the RBF for domestic water supply, considering the removal of microcystins in the Murray River in Australia. They briefly reported on the decomposition of cyanobacterial hepatotoxin microcystins in porous media.
In RBF systems, removing organic pollutants is an important task when water is used for domestic purposes. Absorption and colonization of an organic pollutant could reduce its transfer. Kim et al. (2002) used a kinetic model for the simulation of the fate and transport of dissolved organic pollutants and bacteria. They modeled the porous media using four phases: two colloidal phases, one aquatic phase, and a solid phase. The result of this study shows that the transfer of pollutants in the vicinity of dissolved organic matter is considerably high.
Schon (2006) studied the RBF systems in Austria and India. He concluded that the layout and arrangement of pumping wells based on morphology have a considerable effect on the extension of the treatment area in wells located on the inside of a meander, and the filtration from the riverbank has more sensitivity than wells located outside the meander.
Abdel Fattah et al. (2007) used tracer techniques to trace and evaluate the transfer of water through the alluvial aquifer in El Paso, Texas. They conducted several simulations to show the effects of well's locations and its pumping rate on the flow path, travel time, pumping radius of influence, and ratio of the volume of water from the river to the volume of water from the

aquifer. Also, they found that the pumping rate has more influence on travel time than the distance between the well and the river.
Shamrukh et al. (2008) investigated the effectiveness of the RBF system in Upper Egypt in the Nile valley for removing particulates, dissolved solids, and microbial pathogens to produce drinking water. For this purpose, they monitored physical, chemical, and microbial measurements. They compared water produced with surface and background natural groundwater and with the RBF system and proved the effectiveness of the RBF technique for potable water supply requiring any further treatment or as pre-treatment for higher water quality in Upper Egypt.
Sandhu et al. (2011) studied the operating bank filtration sites in India and investigated the potential of RBF sites based on water problems and hydrogeological suitability. They found that bank filtrate showed higher quality in RBF water compared to water from surface or groundwater sources. They investigated the different uses and the consequent effect on the quality and quantity of surface and groundwater. They stated the RBF system with an emphasis on the hydrogeological conditions, system capacities, and the main water quality improvements. However, they resulted there are some prospects and limitations for the application of bank filtration in India at the existing sites.
Lee et al. (2011) studied using a radial collector well for taking out a large amount of groundwater in a way that hasn't seen a deep drawdown at the well's center. They investigated the hydraulic interaction between river water and groundwater flow response to pumping the riverbank filtration system in Daesan Myeon, Korea. They performed steady-state and transient simulations to estimate the well yield and well responses to pumping. They also evaluated the effect of well structure on the capacity of the RBF well. They resulted in increasing the length of horizontal arms increases the amount of induced river water, and with increasing pumping rates, the effect of well design is more noticeable.
Prasad et al. (2016) presented an optimization model using genetic algorithms for optimal well distance from the river with minimizing the cost of pumping and treatment. The total suspended solids, endosulfan concentrations, and E. coli were considered as water quality parameters. The total suspended used as microbial contamination and are considered not to be absorbed in the aquifer, and endosulfan is considered to undergo sorption. Also, Sensitivity analysis has been done and resulted in the optimal distance significantly affected at lower hydraulic conductivity values, and the cost of treatment increased with increasing hydraulic conductivity. It was concluded that the hydraulic conductivity of the adjoining aquifer plays a dominant role in deciding the optimal distance of pumping wells in a river bank filtration system.
Mustafa et al. (2024) introduce a 3D analytical model utilizing the Green's function approach to analyze the movement of contaminants from the river towards the extraction well within RBF systems. By accounting for the dynamic interaction between river width and the clogging layer, this model offers a more accurate depiction of contaminant transport in three-dimensional water flow scenarios.
Uwimpaye et al. (2025) assessed the suitability of RBF in regions with limited access to clean water, such as Africa, where it has the potential to alleviate water scarcity and enhance water

security. This study used various studies, highlighting the principles, applications, and advancements of RBF worldwide. The findings of this research revealed that RBF effectively addresses a broad range of contaminants, including microbial pathogens, organic compounds, heavy metals, and micro-pollutants, through natural processes like adsorption, biodegradation, and filtration.

As presented in the literature review, little attention has been paid to using mathematical (optimization) models in designing and analyzing RBF systems. This study undertook this task. Importantly, attempts were made not to give details of the chemical, physical, and biological behavior of pollutants; instead, the governing equations of groundwater flow and contaminant fate and transport were developed here.

Typically, an RBF system's extended area is small compared to the total extension of the aquifer. Thus, the aquifer was assumed homogeneous, and hence the use of lumped models and analytical solutions is reasonable. In the case of inhomogeneous aquifers, the methodology may extend to using numerical methods and distributed models, though the solution of the resulting model is more complicated, usually requiring the utilization of a simulation-optimization scheme to achieve the desired results.

In addition to aquifer homogeneity, other major assumptions in this study included the following: the river discharge is permanent; river and wells are fully penetrated; the river and adjacent aquifer are hydraulically connected and there is no low permeable layer on the bed of river; groundwater flow in porous media is Darcian and Dupoit assumption is applicable; drawdowns in comparison with the initial saturated layer is small; and the system design life cycle and interest rate are given. The river pollutant is a single species and Fickian, and not in the form of biological species (microorganisms) or NAPL. Hence, the advection-dispersion solute transport equation governs here; the flow and solute transport from the river to the aquifer is considered one-dimensional, and the linear sorption mechanism has been considered along with the equilibrium chemical reaction. Finally, assuming that the changes in the solute concentration yielded by the solution of the transport equation cause negligible variations in water density, the flow equation and solute transport equation can be solved independently (Zheng and Bennet, 2002).

3. Methodology

3.1. Model Formulation: Steady–State Conditions

3.1.1. Flow Equations

The flow equations of an RBF system in steady-state conditions are derived by combining the continuity equation, Darcy's law, and the image wells concept. Under steady-state conditions, the total water pumped in wells comes from the river. As demonstrated in Fig. 3, the drawdown at distance r from the well pumping equals (McWhorter and Sunada, 1977):

$$s = \frac{Q_w}{2\pi T} Ln \frac{r_i}{r} \quad (1$$

Where, Qw is the constant pumping rate from the well, T represents the aquifer transmissivity, r shows the distance from the river, and ri shows the distance from the image well. Also, the drawdown in the pumping well is derived by incorporating r=rw in Eq. (1).

Fig. 4 indicates a schematic arrangement of the wells near a permanent river as an RBF system. Using Eq. (1) and assuming that the aquifer behaves as a linear system, the required equation could be derived. The drawdown in each well of the system *s(k)* equals the sum of drawdowns due to individual well pumping. Thus, with $N_w$wells, the drawdown *s(k)* in well *k*, equals:

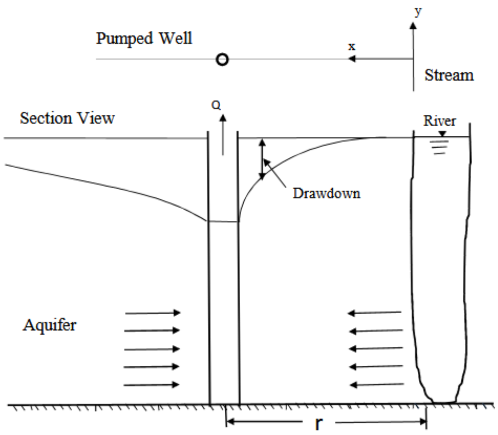

Figure 3. Section view of the system with major components

$$s(k) = \sum_{j=1}^{N_w} s_{k\,j} \quad (2$$

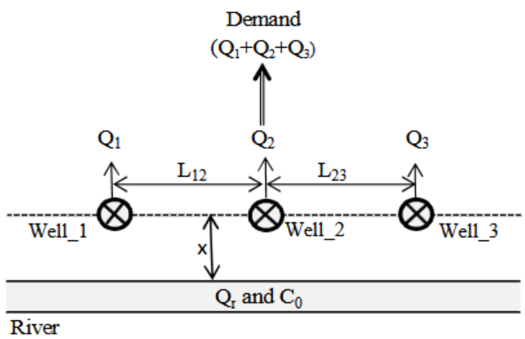

Figure 4. A schematic arrangement of wells near a permanent river

Where, *s(k,j)* is the portion of drawdown in well *k* due to pumping in well *j* (*j* may equal *k*). From Eq. (1), we have:

$$s(k,j) = \frac{Q(j)}{2\pi T} Ln(\frac{\sqrt{4x^2 + l_{kj}^2}}{l_{kj}}) \quad (3$$

Where, $Q_w(j)$ is the constant pumping rate in well *j*, l_{kj} represents the distance between wells *k* and *j* (for *k=j*, $l_{kj}=r_w$, where r_w is the well radius), and *x* denotes the distance between the river and wells' alignment (see Fig. 7).

3.1.2. Solute Transport Equations

The governing equation of the solute transport in porous media with sorption and decay was derived by combining the continuity equation, Fick's first and second laws, and the sorption plus decay mechanism. The resulting equation was a variant of the advection-dispersion equation. Assuming a linear sorption isotherm and one-dimensional unidirectional flow in homogeneous isotropic porous media and first-order irreversible rate reaction (decay rate), we have (Zheng and Bennet, 2002):

$$R_d \frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} - V_x \frac{\partial C}{\partial x} - \lambda C \quad (4$$

Where, Rd represents the retardation factor, C is the solute concentration, t shows the time, Dx is the dispersion coefficient, Vx denotes seepage velocity in the x direction, and λ is the decay rate. From Darcy's law, we have:

$$V_x = \frac{q_x}{\theta} = \frac{K_x}{\theta} i = \frac{K_x}{\theta} \frac{\bar{s}}{x} \quad (5$$

Where, q_x is Darcy velocity or groundwater flux in the *x* direction, K_x shows the hydraulic conductivity in the *x* direction, $\bar{s} = 1/N_w \times \sum_{k=1}^{N_w} s_k$ is the average drawdown of wells, and *x* is the well's distance to the river. The Retardation factor, R_d, is a function of distribution coefficient (K_d) along with the porosity of the aquifer (Θ) and bulk density of the aquifer material (ρ_b), as (Zheng and Bennet, 2002):

$$R_d = 1 + K_d \frac{\rho_b}{\theta} \quad (6$$

The dispersion coefficient D_x is also a function of longitudinal dispersivity (α_L), groundwater velocity (V_x), and molecular diffusivity (D_*) as:

$$D_x = \alpha_L \times V_x + D_* \approx \alpha_L \times V_x \quad (7$$

The longitudinal dispersivity, α_L, is a very uncertain parameter (Gelhar, 1993). However, a well-known equation proposed by Neuman (1990) is applied in the case of data inadequacy as a rough estimate:

$$\alpha_L = 0.0175 \quad L^{1.46}, \qquad 100 \ m < L < 3500 \ m \quad (8\text{-a}$$

$$\alpha_L = 0.0169 \quad L^{1.53}, \qquad L < 100 \quad m \quad (8\text{-b}$$

In which, *L* is the distance between the contaminant source (river in this case) and contaminant exposure (well in this case). As RBF systems are constructed next to the rivers, thus, Eq. (8-b) is applicable to these systems. Using proper initial and boundary conditions, the analytic solution can be conducted. The initial and boundary conditions in this case are:

$$C(x,0) = 0 \qquad x \geq 0 \quad \textit{initial condition} \quad (9$$

$$C(0,t) = C_0 \qquad t \geq 0 \quad \textit{boundary condition} \quad (10$$

$$C(\infty,t) = 0 \qquad t \geq 0 \quad \textit{boundary condition} \quad (11$$

$$\frac{\partial C(\infty,t)}{\partial x} = 0 \qquad t \geq 0 \quad \textit{boundary condition} \quad (12$$

Where, C_0 is the constant concentration of pollution in the river (*mg/lit*) and *C(x,t)* represents the (reduced) concentration of leaked water in the riverbank at distance *x* after time *t*. The analytical solution for this problem has been given by Batu as (Batu, 2006):

$$C(x,t) = \frac{C_0}{2}\left[\begin{array}{l} erfc\left[\frac{R_d x - (V_x{}^2 + 4D_x R_d \lambda)t}{2(D_x R_d t)^{0.5}}\right] \times exp(\frac{(V_x - (V_x^2 + 4D_x R_d \lambda))x}{2D_x}) + \\ erfc\left[\frac{R_d x + (V_x{}^2 + 4D_x R_d \lambda)t}{2(D_x R_d t)^{0.5}}\right] \times exp(\frac{(V_x + (V_x^2 + 4D_x R_d \lambda))x}{2D_x}) \end{array}\right] \quad (13$$

Under steady-state conditions ($t\rightarrow\infty$), the above equation is reduced to:

$$C(x) = C_0 \times exp\left[\frac{x(V_x - (V_x{}^2 + 4D_x R_d \lambda))}{2D_x}\right] \quad (14$$

In which, *C(x)* is the well water concentration at a distance *x* from the river (*mg/lit*).

3.2. Optimization Model

Assuming that the river discharge and its pollution concentration, as well as wells' pumping rates and the resulting drawdowns, are all constant, the steady-state optimization model can be developed. As displayed in Fig. 5, let C_0 be the constant concentration of pollution, *x* the wells' distance to the river, l_{kj} the distance between wells *k* and *j*, $Q_w(k)$ the discharge of well *k* and *Q* the required demand (*Q=Dem*). Then, the objective function of this cost-effective problem is:

$$Min \ \ Z = C_{Inst}^{w} + C_{Pump}^{w} + C_{Inst}^{p} + C_{conv}^{p} + C_{treat} \quad (15$$

Where, C_{inst}^{w} is the cost of installing wells (such as construction of wells, casing, pumps purchased, installation, etc.), C_{pump}^{w} represents the pumping cost of the well, which is a function of the well's discharge ($Q_w(k)$) and the well's total lift height (h_k),C_{inst}^{p} denotes the installation costs of pipelines (construction, purchasing, installation, etc.), C_{conv}^{p} is the cost of the pipeline's water conveyance, which is a function of the well's discharge and distance, and finally C_{treat} shows the cost of treatment of water due to residual concentration (pollution) remaining in the well water. Then, C_{inst}^{w} and C_{inst}^{p} represent the operation and maintenance cost since this cost is usually considered as a fraction of the initial capital cost. All costs are considered in equivalent annual form. C_{pump}^{w} equals to:

$$C_{pump}^{w} = \sum_{k=1}^{Nw} C_{pump}^{w}(k) \quad (16$$

Where, N_w represents the total number of wells and $C_{pump}^{w}(k)$ is the annual pumping cost of well *k*, which is equal to:

$$C_{pump}^{w}(k) = Hour(k) \times UCE \times P_w(k) \quad (17$$

Where, *UCE* is the unit cost of energy *($/kWh*) and $P_w(k)$ is the power utilized for the pump in well *k*. *Hour(k)* denotes the hours that pump *k* works in a year. Also,

$$p_w(k) = \gamma \times Q_w(k) \times (h^{ini}(k) + s(k))/\varepsilon \quad (18$$

Where, γ is the specific weight of water ($9806\ N/m^3$), $h^{ini}(k)$ is the initial lift (distance between ground surface and groundwater table before pumping) of well k, $s(k)$ represents the drawdown in well k in response to pumping of all wells (Eq. (2)), and ε is the pump's overall efficiency. The conveyance cost equals:

$$C_{conv}^{p} = UCE \times \sum_{k=1}^{Nw} Q_w(k) \times (\Delta H + H_l) \times Hour(k) \quad (19$$

Where, ΔH is the elevation difference between the collector pipe and pumping house (Fig. 2) and H_l represents the total head loss (friction and local head losses) in pipes. Based on the Darcy-Weisbach equation, the friction head loss (H^{frc}) is a function of the pipe's length, diameter and discharge. Similarly, the local head loss (H^{loc}) is a function of the pipe's diameter and discharge (Chin, 2012). Thus:

$$H_l = \sum_{k=1}^{Nw-1} H_{kj}^{frc} + H_{coll}^{frc} + H_{coll}^{loc}; \quad \text{j} = k+1 \quad (20$$

$$H^{frc} = f\frac{16lQ^2}{\pi^2 D^4} \quad (21$$

$$f = \frac{0.25}{\left[\left[log(\frac{k_s}{3.7D}\right] + \frac{5.74}{Re^{0.9}})\right]^2} \quad \left(\text{for } 10^{-6} \le \frac{k_s}{D} \le 10^{-2}, \quad 5000 \le \text{Re} \le 10^8\right) \quad (22$$

$$Re=VD/\upsilon=4Q/(\pi D\upsilon)=1273240Q/D \quad (23$$

$$H_{coll}^{loc} = K\frac{16l_{coll}Q_{coll}^2}{\pi^2 D_{coll}^4} \quad (24$$

Where, f is the pipe friction factor, k_s shows the pipe roughness, l is the pipe length, Q denotes the pipe discharge, D indicates the pipe diameter, and Re is the Reynolds number. Equations 21-23 must be written for each pipe (using l_{kj}, Q_{kj} , D_{kj} , f_{kj}, and Re_{kj} for each pipe k-j located between wells k and j, as well as using l_{coll}, Q_{coll} , D_{coll} , f_{coll}, and Re_{coll} for the collector pipe). Also, V is the pipe water velocity and υ shows the kinematic viscosity of water (equals 10^{-5} m^2/s at 20^oC). Note that $Q_{coll} = \sum Q_w(k) = Dem$ and Q_{kj} is related to the pipe k-j position in the system. For example, in the 5-well configuration of Fig. 7, by considering that the collector pipe is located back of the well 3, we have $Q_{12}=Q_w(1)$, $Q_{23}=Q_w(1)+Q_w(2)$, $Q_{34}=Q_w(4)+Q_w(5)$, and $Q_{45}=Q_w(5)$. Thus,

$$\underset{(j=k+1)}{Q_{kj}} = \begin{cases} Q_w(1), & forthe\ firstpipe \\ \sum_{k=1}^{\left[\frac{N_w-1}{2}\right]} Q_w(k)\,, forthe\ middle\ pipes\ between\ well 1\ and\ well\ [(N_w-1)/2]; & \\ \sum_{k=[(N_w+1)/2]}^{N_w} Q_w(k)\,, for\ the\ middle\ pipes\ between\ well \left[\frac{N_w+1}{2}\right] and\ well\ N_w; & \\ Q_w(N_w), & forthe\ lastpipe \end{cases} \quad (25$$

Where, [u] is the integer value of u.

Obviously, the water treatment cost increases by the elevating the water volume (*Dem*) and solute concentration (C), as more chemicals are needed to reduce the concentration up to the standard levels. Thus, this term is a function of the remaining concentration *C(x)* (as depicted in Eq. (13)) and the volume of water that would be treated.

$$C_{treat} = f_1(C(x), Dem) \quad (26$$

The main attractive feature of RBF systems is the property of removing large amounts of pollutants in flowing river water. Indeed, most of the river water pollutants that enter the aquifer and then the wells can be removed through porous media. However, some pollutants remain in the wells from which removing these residual pollutants is necessary to achieve standard levels. There are various treatment methods for removing residual pollutants in water, and here the disinfection with Chlorine and Chloramine (Wilbert et al. 1999) has been used as a common method of treatment. Cost estimation for Chlorine and/or Chloramine disinfection is based on the amount of chemicals used per day. Chlorine demand is determined from the concentration of nitrite and reduced inorganic transition metals, including chromium, copper, iron, and manganese present in the water (Wilbert et al. 1999). The detailed relations of disinfection by chlorine and ammonia have been given by Wilbert et al. (1999), and its companion, a Microsoft Excel file (WaTER). The treatment cost with this method is a multivariate function:

$$C_{treat} = f_2(D, C(x), DCR, AAD, Cl_2Cost, AmmCost) \quad (27$$

Where, *Q* is the production flow rate to be treated (*Q=Dem*), *C(x)* represents the pollutant concentration in water, *DCR* denotes the desired chlorine residual, *AAD* is the alternative ammonia dose, Cl_2Cost shows the cost of Cl_2 and *AmmCost* indicates the cost of NH_4OH. The product flow rate (*Q*) and porous media characteristic (*Θ, S, T*) are known, but the pollution concentration in the well water is unknown. The farther the distance to the river, the more natural the treatment in porous media will be, thus reducing treatment costs. Also, more drawdown in wells increases the pumping cost. Thus, there is an optimum distance that minimizes the total cost of treatment and pumping.

Finally, the required demand (*Dem*) must be met. Thus,

$$\sum_{k=1}^{N_w} Q_w\,(k) = Dem \quad (28$$

For a given demand (*Q=Dem*) and number of wells (N_w), C_{Inst}^{w} and C_{Inst}^{p} are constant and do not alter the model result.

3.3. Model Formulation; Transient Conditions

Unlike the steady-state conditions, in transient conditions, here the variability of river discharge and solute concentration, well pumping, and the obtained drawdowns were considered. To develop transient flow and solute transport equations, the concept of linear systems theory was applied. The Cooper-Jacob equation for computing drawdown of wells (Todd and Mays, 2005), the image wells concept for computing river aquifer interaction, and Eq. (12) for computing the solute concentration were used under transient conditions. The linear systems theory was used to derive the required equations in variable pumping as well as variable solute concentration.

3.4. Response Functions of Linear Systems

The linear systems are those whose behavior may be predicted by a linear differential equation. Linear systems have two basic properties, which are proportionality and additivity. Indeed, the response function of a linear system is a solution of its governing differential equations. In many situations, such as computing drawdown or solute concentration in an aquifer, the governing equation is indeed linear; thus, the rules of linear systems can be used in deriving the desired relations.

If a system receives an input of unit value applied instantaneously (a unit impulse) at time τ, the response of the system at a later time t is described by the unit impulse response function $u(t-\tau)$. The response to the complete input time function $I(\tau)$ can then be obtained by integrating the response to its constituent impulses, which is called the convolution integral (Chow et al. 1988):

$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau \tag{29}$$

A unit step input response function $g(t)$ is found from (29) with $I(\tau)=1$ for $\tau \geq 0$ as:

$$g(t) = \int_0^t u(t-\tau)d\tau = \int_0^t u(l)dl \tag{30}$$

Also, the unit pulse response function $h(t)$ equals" (Chow et al. 1988):

$$h(t) = \frac{1}{\Delta t}[g(t) - g(t-\Delta t)] \tag{31}$$

If Δt set equals unity ($\Delta t=1$), then:

$$h(t) = g(t) - g(t-1) \tag{32}$$

Further, for $t<0$, $g(t)=0$. The drawdown derived from Theis or Cooper-Jacob equations, assuming $Q=1$ is a unit step response function. If the unit pumping shuts down at the end of $t=1$, the derived drawdown is the unit pulse response function. Fig. 5 displays the unit step and unit pulse response functions of a confined aquifer.

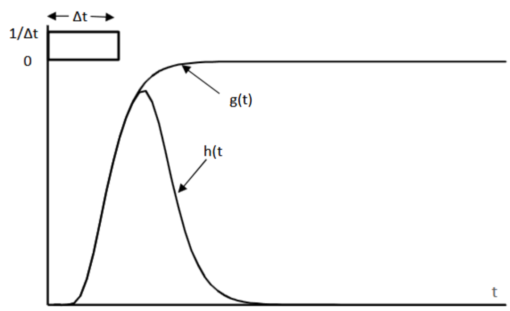

Figure 5. Unit step (*g(t)*) and unit pulse (*h(t)*) response functions of a confined aquifer

3.5. Flow Equation, Transient Condition

The flow equation in RBF systems in transient conditions is required for computing the drawdown of wells and river seepage to each well. These equations, derived by integrating the Cooper-Jacob equation, the concept of image wells, and the unit pulse response function, are as follows:

The drawdown in a confined aquifer due to constant well discharge Q may be derived using the Cooper-Jacob equation (assuming $u = \frac{r^2 S}{4Tt} < 0.01$),

$$s(t) = \frac{Q_w}{4\pi T} ln(\frac{2.246Tt}{r^2 S}) \quad (33$$

Where, *s(t)* represents the drawdown at distance r from the pumping well at time t after the start of pumping, T and S show the aquifer transmissivity and storativity, respectively, and t is the time passed since the beginning of pumping. This equation is linear due to the pumping rate Q_w. By setting $Q_w=1$, the derived drawdown is a unit step response function:

$$g_w(t) = \frac{1}{4\pi T} ln(\frac{2.246Tt}{r^2 S}) \quad (34$$

Following that, the related unit pulse response function yields:

$$h_w(t) = g_w(t) - g_w(t-1) = \frac{1}{4\pi T} ln(\frac{t}{t-1}) \quad (35$$

Note that for $t>1$, the unit pulse response function $h_w(t)$ is independent of space, i.e., the locations of wells and aquifer storativity. Fig. 6 demonstrates $g_w(t)$ and $h_w(t)$ functions for a confined aquifer with $T=1000\ m^2/day$ and $S=0.01$ for different distances from $r=1m$ to $r=100\ m$.

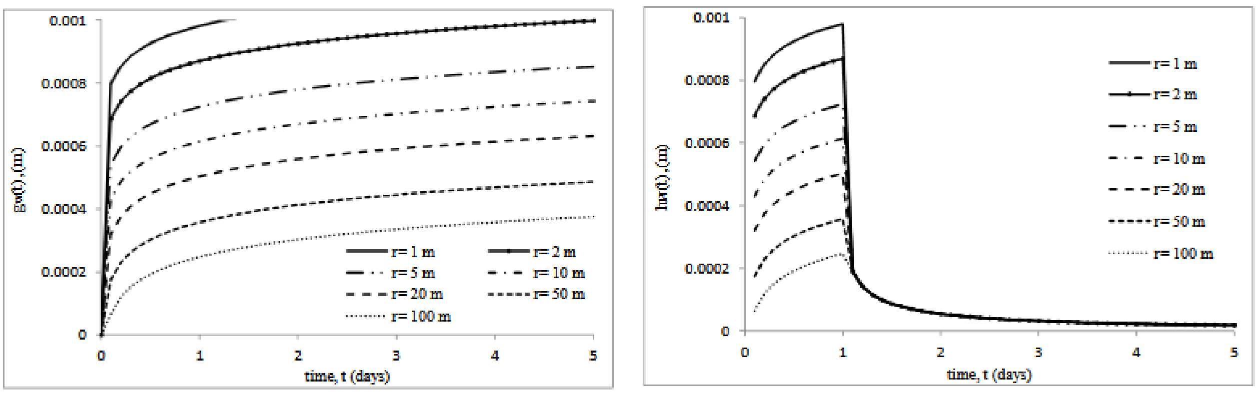

Figure 6. Unit step response ($g_w(t)$) and unit pulse response ($h_w(t)$) functions for a confined aquifer with T=1000 m^2/day and S=0.01

For variable pumping, $Q_w(t)$, the resulting drawdown is derived as:

$$s(n) = \sum_{t=1}^{N} h_w(n-t+1) Q_w(t) \quad (36$$

This is a form of discrete convolution equation for a linear system (Chow et al., 1988).

When a set of N_w wells exists, each with $Q_w(j,t)$, $j=1, 2,..., N_w$, by substituting $h_w(n-t+1)$ by $h_w(k, j, n-t+1)$, which is now called "unit response coefficient" of drawdown of wells, the obtained equation is:

$$s(k,n) = \sum_{t=1}^{n} \sum_{j=1}^{Nw} h_w(k,j,n-t+1) Q_w(j,t) \qquad (37$$

This equation is known as the unit response matrix method, which was first introduced in groundwater systems optimization by Maddock (1972). Alimohammadi et al. (2009) later developed general equations for deriving response equations for point, linear, and surface excitations in aquifers.

When there are some sources, such as return flows or river leakage, these sources should also be considered. In RBF systems, river leakage exists and thus this river-aquifer interaction must be considered. One approach is to use the concept of image wells. Fig. 7 illustrates a series of pumping wells (*k,j,…*) near a permanent river and its corresponding image wells (*k',j',…*). The drawdown in well *k* due to constant pumping (Q_w) in well *j* based on the concept of image wells and Cooper-Jacob equation equals:

$$s_r(t) = \frac{Q_w}{4\pi T}\left[ln(\frac{2.246Tt}{r^2 S}) - ln(\frac{2.246Tt}{r_i^2 S})\right] = \frac{Q_w}{2\pi T} ln(\frac{r_i}{r}) = \frac{Q_w}{2\pi T} ln(\frac{L_{kj'}}{L_{kj}}) = Cte. \qquad (38$$

$$L_{kj'} = \begin{cases} 2x, if j' = k' \\ \sqrt{L_{kj}^2 + 4x^2}, if j' \neq k' \end{cases} \qquad (39$$

Where, r and r_i represent the distance from the pumping well and image well, respectively. Also, $L_{kj}=r_w$ for $k=j$. Eq. (38) is the same as Eq. (1). Note that $s_r(t)$ is independent of time. This condition is known as pseudo steady-state conditions (McWhorter and Sunada, 1977). In Eq. (38), Q_w is constant, and the equation is linear due to Q_w. Therefore, the properties of linear systems could be used again in this case. By setting $Q_w=1$, the derived net drawdown is a unit step response function as:

$$g_r(t) = \frac{1}{2\pi T} ln(\frac{L_{kj'}}{L_{kj}}) \qquad (40$$

Also,

$$h_r(t) = \begin{cases} g_r(t), \text{for t} \leq 1 \\ 0, \text{for t} > 1 \end{cases} \qquad (41$$

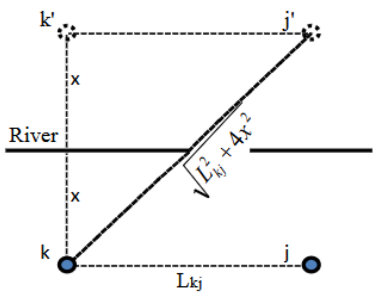

Figure 7. Pumping wells (*k,j,..*), and corresponding image wells (*k',j',…*)

For a set of N_w wells, we have:

$$g_r(k,j,t)=g_r(k,j)=\frac{1}{4\pi T}\left[\sum_{j=1}^{Nw} ln(\frac{2.246Tt}{L_{kj}^2 S})-\sum_{j'}^{Nw} ln(\frac{2.246Tt}{L_{kj'}^2 S})\right]=\frac{1}{4\pi T}\left[\sum_{j=1}^{Nw} ln(\frac{L_{kj'}^2}{L_{kj}^2})\right]=$$

$$\frac{N_w}{2\pi T} ln\left(\frac{\prod_{j'=1}^{Nw} L_{kj'}}{\prod_{j=1}^{Nw} L_{kj}}\right) \qquad (42$$

The equation for net drawdown due to variable pumping rates $Q_w(j,t)$ of N_w wells is obtained as:

$$s_r(k,n)=\sum_{t=1}^{n}\sum_{j=1}^{Nw} h_r(k,j,n-t+1)Q_w(j,t) \qquad (43$$

Based on Eqs. (41) and (42), it is reduced to:

$$s_r(k,t)=\frac{Nw}{2\pi T} ln\left(\frac{\prod_{j'=1}^{Nw} L_{kj'}}{\prod_{j=1}^{Nw} L_{kj}}\right)\sum_{j=1}^{Nw} Q_w(j,t) \qquad (44$$

3.6. Solute Transport Equation-Transient Conditions

The term transient here refers to the difference in the river solute concentration. According to Eq. (13), the equation is linear due to the constant concentration C_0. Thus, as with flow equations, in this case, we have:

$$g_c(x,t) = \frac{1}{2}\begin{bmatrix} erfc\left[\frac{R_d x-({V_x}^2+4D_x R_d \lambda)t}{2(D_x R_d t)^{0.5}}\right]\times exp(\frac{(V_x-(V_x^2+4D_x R_d\lambda))x}{2D_x})+ \\ erfc\left[\frac{R_d x+({V_x}^2+4D_x R_d \lambda)t}{2(D_x R_d t)^{0.5}}\right]\times exp(\frac{(V_x+(V_x^2+4D_x R_d\lambda))x}{2D_x}) \end{bmatrix} \qquad (45$$

$$h_c(x,t)=g_c(x,t)-g_c(x,t-1) \qquad (46$$

Where, $g_c(x,t)$ is the unit step response function and $h_c(x,t)$ is the unit pulse response function or unit response matrix of the river leakage, and:

$$C_c(x,n)=\sum_{t=1}^{n} h_c(x,n-t+1)\times C_0(t) \qquad (47$$

Where, $C_c(x,n)$ represents the solute concentration in the well within time period n, and $C_0(t)$ is the river solute concentration within time period t. Similarly, for well j within time period t, we have:

$$C_c(x,j,n)=\sum_{t=1}^{n} h_c(x,j,n-t+1)\times C_0(t) \qquad (48$$

Where, $h_c(x, j, n\text{-}t+1)$ is the unit pulse response function or unit response matrix of the solute concentration for well j within time period $n\text{-}t+1$.
The optimization model in transient conditions is similar to the steady-state model, but using unsteady terms such as $Q_w(k,t)$ and $C_c(x,t)$ instead of $Q_w(k)$ and $C_c(x)$, as well as flow and transport formulation in transient conditions.

4. Results

In this section, two numerical examples have been presented. The first example illustrates the proposed formulation for the design of a hypothetical RBF system. In the design problem, the steady-state conditions, with some critical situations such as river discharge equal to $_7Q_{10}$ (7-day minimum discharge with 10 years return period) and high pollutant concentration, could be considered. The second example represents the analysis of an existing RBF system in a 12-month period. Obviously, for this example, transient conditions must be considered.

4.1. Example Problem 1, Design of an RBF System

A hypothetical example has been considered here to show how the developed proposed model works in the case of design problems. A small city located next to a relatively large permanent river has been considered. Its municipality has a plan for designing an RBF system as a backup water supply system. The design discharge (required demand) equals 0.5 m^3/s. The river's $_7Q_{10}$ discharge is far larger than the design discharge. Five wells have been considered in this plan, with Fig. 7 demonstrating a schematic view of this problem. The radius of the wells equals 0.5 *m,* and the distance between the ground surface and the static groundwater table equals 10 *m*. The aquifer's transmissivity and porosity equal 0.01 m^2/s and 0.3, respectively. The pumping house is located 20 *m* away from the collector pipe and in 10 *m* upper elevation that has required as the local network head. Other data on the problem are reported in Tables 1 and 2. Table 2 shows the parameters of disinfection treatment and piping facilities. Based on construction limitations, equal sizes have been considered here for the diameter of all collector pipes of the wells. The entire optimization model formulation and solution have been implemented in Microsoft Excel® as a spreadsheet model. Table 3 shows the problem solution results using the Solver add-on of Microsoft Excel®.

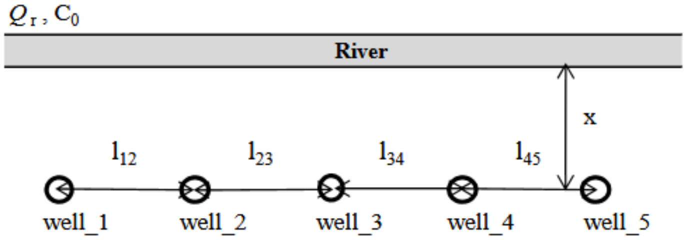

Figure 8. A schematic view of the example problem 1

Table 1. Input parameters

Parameter	Unit	Value
Demand	m^3/s	0.5

Wells radius	m	0.5
Transmissivity	m^3/s	0.05
Porosity	-	0.3
Saturated thickness	m	50
Initial Lift	m	10
Initial concentration	Mg/liter	500
Minimum well distance	m	20
Maximum well distance	m	100
Retardation factor	-	20
Decay rate	-	0.000001
Plan duration	Day	7
Unit cost of energy	4/Kwh	0.00714

Table 2. Parameters of disinfection treatment and pumping facilities

Parameter	Units	Value
Residual chloramines	mg/L	3
Alternative chlorine dose	mg/L	6
Alternative ammonia dose	mg/L	2
Cl_2 needed	mg/L	21.18
Ammonia needed	mg/L	0.99
Cl_2 unit cost	$/ton	50
Basis ammonia	Kg/day	172.8
NH_4OH unit cost	$/ton	250
Plant availability	-	0.95
Collector-motor house distance	m	20
Collector-motor house ΔH	m	10
Pipes roughness	m	0.00015
Pipe velocity	m/s	3

Table 3.The problem solution brief results

Well#	1,5	2,4	3
Discharge (l/s)	68.7	97.6	167.3
Drawdown (m)	1.45	2.54	3.54
Distance from center (m)	85.0	22.7	0
Used power (Kw)	9.65	15.01	27.78
Used energy (Mwh)	1.621	2.522	4.667
Pumping cost ($)	11.577	18.012	33.339
X=36.7 m C=34.10 mg/lit	α_L=4.18 D_x=0.000876 m2/s		
Pipe diameter=12 in. (0.3048 m)*			
Collector diameter=18 in. (0.4572)			
Total pumping cost= 93 $			
Total conveyance cost= 91 $			
Treatment cost=98 $			
Total cost=281 $			

*: Equal sizes have been considered here for all pipe diameters.

(1 inch=2.54 cm)

As presented in Table 3, the results of the model show that in this case, the well alignment should be located 36.7 *m* away from the river. The wells' discharges and distances are symmetric where $Q_1=Q_5$, $Q_2=Q_4$, $l_{12}=l_{45}$, and $l_{23}=l_{34}$. The central well has more while the side wells have less discharge. Also, the pollutant concentration in wells decreases to 34.1 mg/L from the initial 500 mg/L in the river. Unexpectedly, the discharge of the central well is greater than and that of side wells is less than discharge of other wells. Since both pumping and conveyance costs have been considered here, the conveyance cost (which grows with distancing off the wells) outweighs the pumping cost (which decreases with farther distance off the wells) here. This suggests that if only the pumping cost has been considered, opposite results can be expected. Three cost terms (pumping, conveyance, and treatment) approached each other in this case, suggesting that none of them is ignorable.

4.1.1. Sensitivity Analysis

A local sensitivity analysis has been implemented here for assessing the relative importance of the model parameters. Five parameters, including unit cost of energy (*UCE*), retardation factor (R_d), transmissivity (*T*), porosity (Θ), and decay rate (λ), have been considered. Fig. 9 displays the variations of variable *x* against the variations of the five above-mentioned parameters. As can be seen, the variations of *x* versus *T* are ascending, but for other parameters, they are decreasing. With the elevation of *T*, the value of *K* has grown (the thickness of the saturated layer was considered constant). Thus, the seepage or solute transport velocity increases, while for a given concentration, the value of *x* drops. The variable *x* is more or less sensitive to all parameters, with the sensitivity being greater on the left-hand side (values of parameters that are less than initial values) than on the right-hand side (values of parameters that are greater than the initial values). The sensitivity of *T* and *UCE* is almost equal but with opposite directions, and both are less sensitive than the other three variables. R_d, Θ, and λ revealed similar sensitivity.

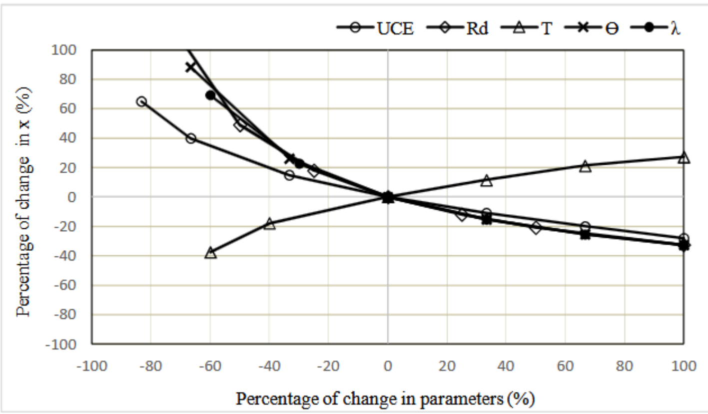

Figure 9. Variations of *x* against the variations of five parameters

Fig. 10 illustrates the variations of variable L_{12} against the variations of the five parameters. L_{12} is not sensitive to variations of *T*, but is sensitive to other variables. In this regard, *UCE* is less sensitive than the other three variables, indicating the same behavior. Again, the sensitivity has been greater on the left-hand side.

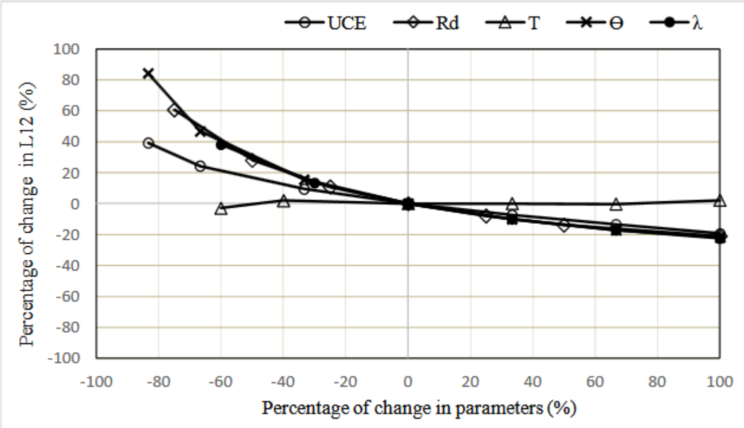

Figure 10. Variations of L_{12} against the variations of five parameters

Fig. 11 reveals the results of sensitivity analysis for concentration *C*. *C* is sensitive to all parameters, especially to *UCE* and *T*. By increasing *UCE*, *x* declines (Fig. 9), thereby augmenting *C*, but by increasing *T*, *x* rises (Fig. 9), thereby lowering *C*. The sensitivity of *C* to R_d, Θ, and λ has been similar, i.e., the variations of these parameters show a similar effect on *C*. Increasing R_d means lengthening retarding and reducing the solute transport velocity, thereby lessening *C*. Also, increasing Θ means diminished seepage velocity and again lower solute transfer velocity. Finally, increasing λ signifies greater contaminant decay and lower concentration in the wells. To assess this further, Fig. 12 reveals the variations of C/C_0 against the variations of Θ/Θ_0, R_d/R_{d0}, and λ/λ_0 where C_0 =500 mg/L, $\Theta_0=0.3$, $R_{d0}=20$, and $\lambda_0=0.000001$. As can be seen, the variations for R_d/R_{d0} and λ/λ_0 are the same (see Eq. (14)), while for Θ/Θ_0, there is a little difference between the two other curves.

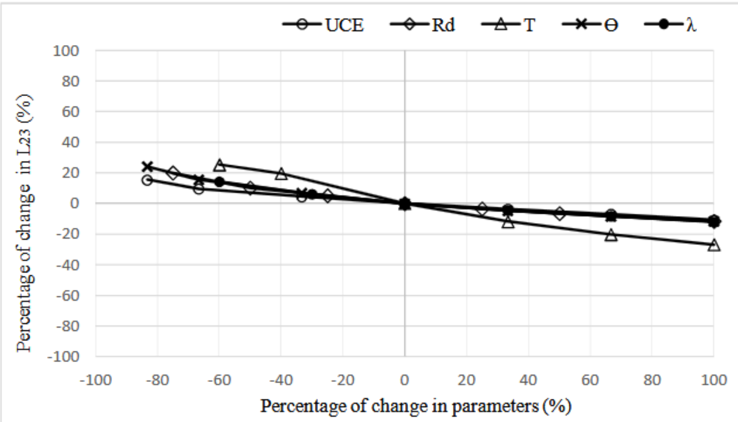

Figure 11. Variations of *C* against the variations of five parameters

Fig. 13 displays the variations of average drawdown in five wells (*s_ave*) against the variations of the five parameters. Drawdown is an inverse function of transmissivity (Eq. (1)), thus is more sensitive to

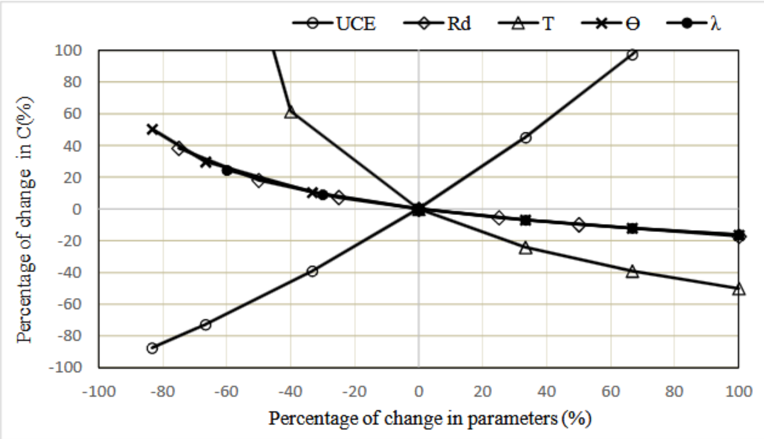

Figure 12. Variations of *C/C₀* against the variations of *Θ/ Θ₀*, *Rd/ Rd₀*, and *λ/ λ₀*

T than to other parameters. In smaller values of parameters, *UCE* is less sensitive than R_d, Θ, and λ (these are solute transport parameters and are not directly related to drawdown, but they directly affect distance *x*. Also, because *s_ave* is related to *x* (*i=s_ave/x* where *i* is the gradient), thus three parameters are indirectly related to *s_ave*.). However, for larger values, *s_ave* is relatively less sensitive to *UCE*, R_d, Θ, and λ.

Fig. 14 indicates the variations of the pumping cost against the variations of the five parameters. Except for *UCE*, other parameters seem to be less sensitive. Since *T* directly affected the drawdown and thus the required energy, it is more sensitive than the other three parameters (which are transport parameters). Also, Fig. 15 exhibits the variations of conveyance cost against the variations of the five parameters. Except for *UCE*, other parameters have been insensitive.

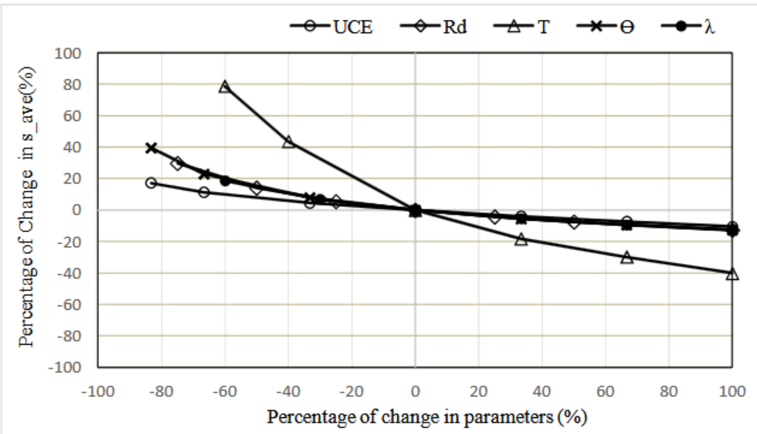

Figure 13. Variations of average drawdown in wells against the variations of five parameters

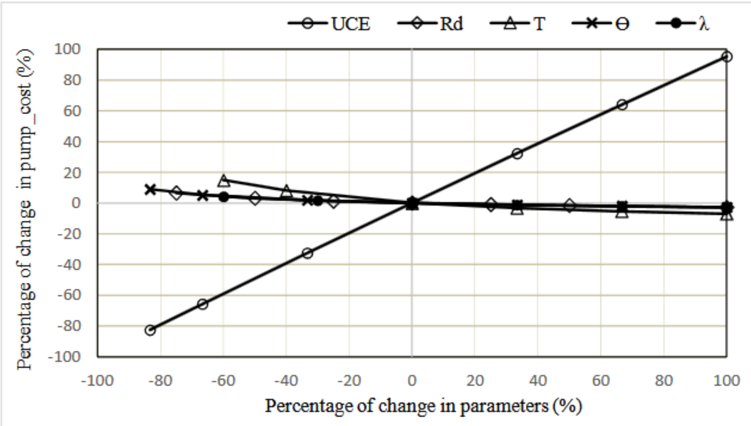

Figure 14. Variations of pumping cost against the variations of five parameters

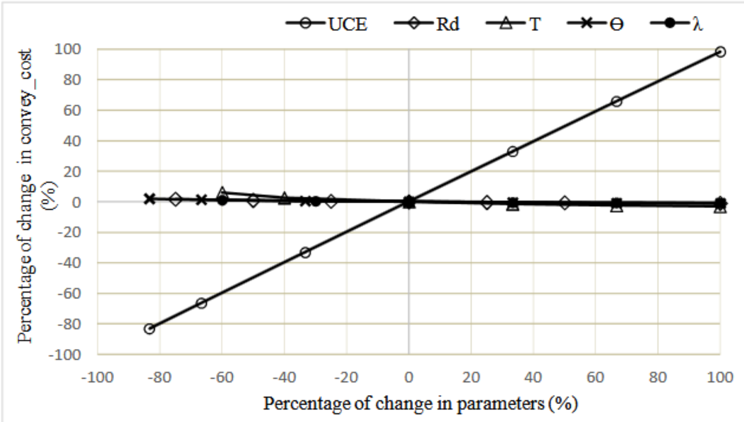

Figure 15. Variations of conveyance cost against the variations of five parameters

Since there are several output variables (*x, C, Costs, …*), for reaching a conclusion on the sensitivity analysis, a dimensionless parameter is defined as:

$$Sens_p = \frac{1}{n_y}\sum_{y=1}^{n_y}\sum_{i=1}^{n_i}\left(\frac{\left|\frac{y_i - y_0}{y_0}\right|}{\left|\frac{x_i - x_0}{x_0}\right|}\right) \quad (49$$

Where, $Sens_p$ is the sensitivity of parameter p, y_i represents the value of variable y ($y \in \{x, C, s, Cost, \ldots\}$) corresponding to the value of parameter x_i ($x \in \{UCE, Rd, T, \Theta, \lambda\}$). Table 4 presents the results of computing Eq. (48). According to this table, *UCE* is a relatively more sensitive while R_d is a relatively less sensitive parameter, but generally all five parameters have been sensitive. Figs. 9-15 are in line with this conclusion.

Table 4. Sensitivity of five parameters

Parameter	Sensitivity	Rank
UCE	0.69	1
Rd	0.30	5

T	0.45	2
ɵ	0.34	4
λ	0.37	3

4.2. Example Problem 2: Analysis of an RBF System

The purpose of this example is to show the ability of the proposed transient formulation in computing the solute concentration in a well next to a river, and the effects of changing parameters on the concentration. A pumping well is located next to a permanent river at a distance $x= 30\ m$. The aquifer and transport parameters are similar to example 1 ($T=0.05\ m^2/s$, $K=0.001\ m/s$, $R_d=20$, $\lambda=10^{-5}$,$\theta=0.3$). In pseudo-steady conditions, the drawdown in the well equals 9 m. Table 5 reports the variations of the concentration of a solute pollutant in the river within a 12-month period (the concentration had been assumed constant through each month). Then, the concentration should be computed in the well water within each period.

Table 5. Concentration of pollution in the river and in the well

Month (t)	C_0(t)	C(t)
1	1000	653.92
2	1300	1008.82
3	1100	928.49
4	800	701.47
5	1000	784.08
6	700	624.83
7	1100	833.29
8	1200	961.33
9	500	520.58
10	1000	736.74
11	800	679.83
12	700	585.67

Using Eq. (45) and considering a computation time step equal to 1 day for greater accuracy, Fig. 16 illustrates the unit step (g_c) and unit pulse (h_c) response functions of the well concentration. Also, Fig. 17 reveals the variations of concentration in the river ($C_0(t)$) and in the well ($C(t)$) in a 12-month period. Attenuation and time lag of concentration in the well's water are clear in the figure.

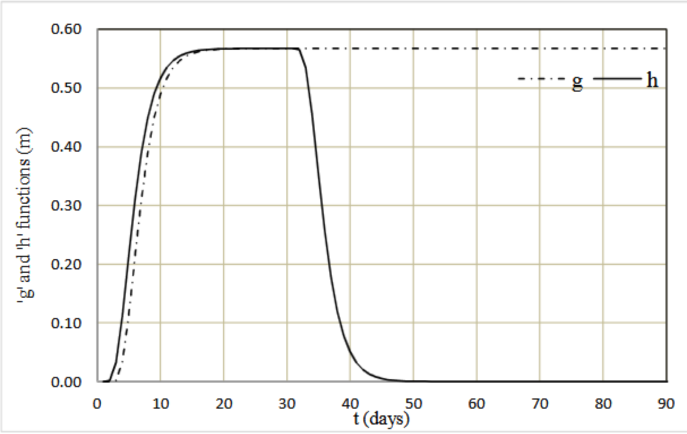

Figure 16. Unit step (g) and unit pulse (h) response functions of the well's concentration

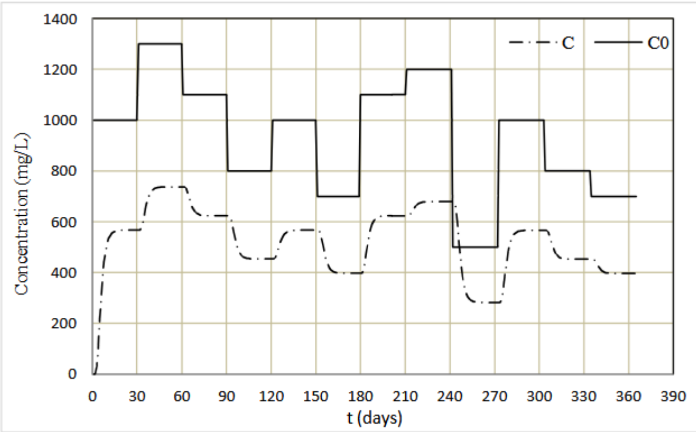

Figure 17. Variations of concentration in the river (*$C_0(t)$*) and in the well (*$C(t)$*)

In the first example, it was observed that Rd, T, Θ, and λ are sensitive parameters of flow and transport models. Here, the effect of these parameters on the variations of concentration and solute transport is observed. Fig. 18 displays the gc and hc functions for different values of Rd. By increasing Rd (retardation), the solute is more retarded and thus gc and hc functions decline. Fig. 19 shows the solute concentration variations for different values of Rd.

Fig. 20 shows the g_c and h_c functions for different values of T. By increasing T (increasing seepage velocity), g_c and h_c functions, as well as concentration, grow. Fig. 21 demonstrates the variations of solute concentration for different values of T.

Fig. 22 shows the g_c and h_c functions for different values of Θ. By increasing Θ (decreasing seepage velocity), g_c and h_c functions as well as the concentration drop. Fig. 23 exhibits the variations of solute concentration for different values of T.

Fig. 24 reveals the g_c and h_c functions for different values of λ. By increasing λ (increasing decay), g_c and h_c functions as well as the concentration fall. Fig. 25 indicates the variations of solute concentration for different values of T. At relatively large values of λ (say λ=10^{-5}), the concentration in the well reaches zero (one method of protecting the well's water is using permeable reactive barriers with nanoparticles with a relatively large λ).

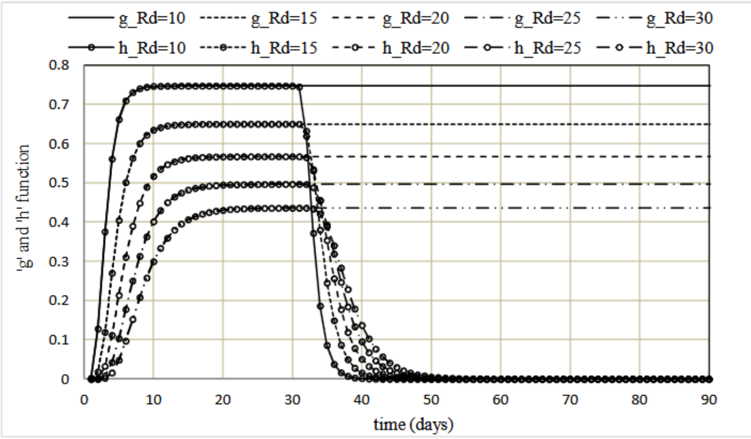

Figure 18. g_c **and** h_c **functions of the well's concentration for different values of** R_d

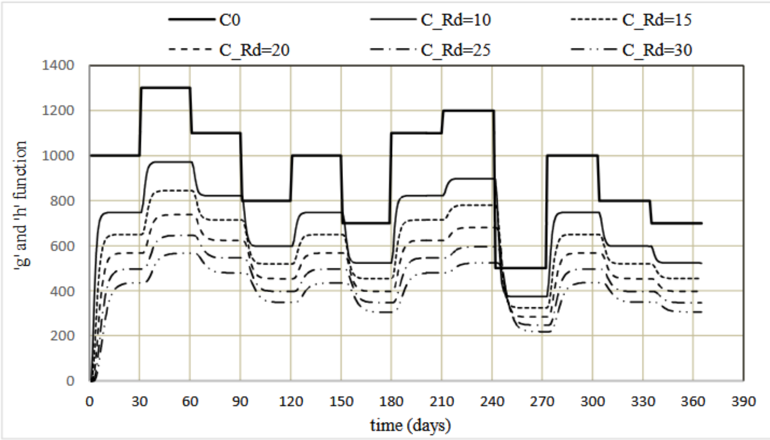

Figure 19. Variations of concentration in the well (*C(t)*) for different values of R_d

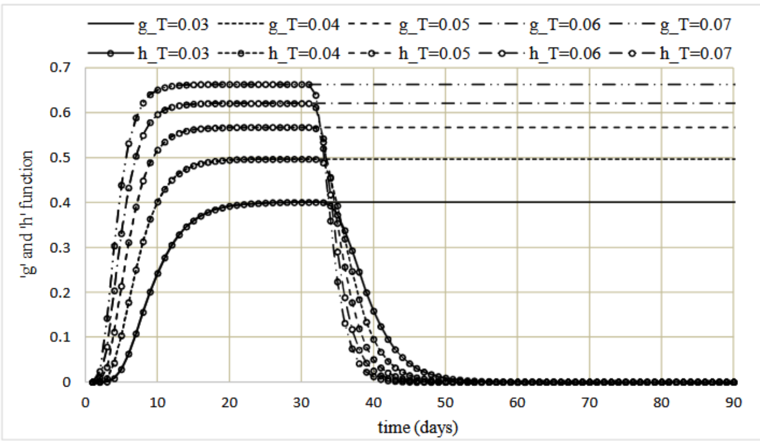

Figure 20. g_c **and** h_c **functions of the well's concentration for different values of** T

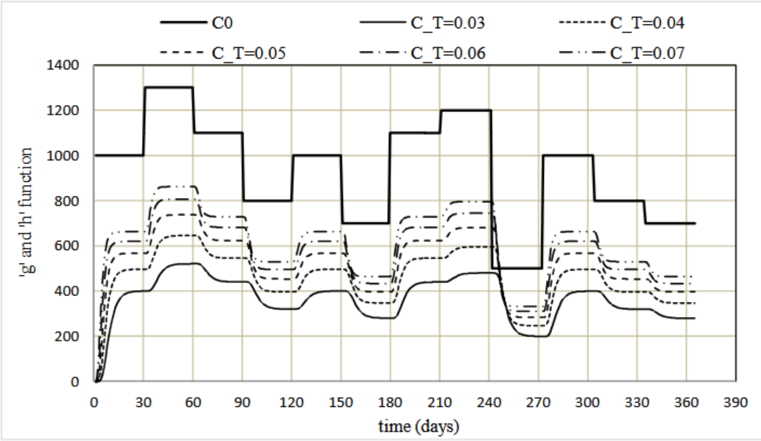

Figure 21. Variations of concentration in the well (*C(t)*) for different values of *T*

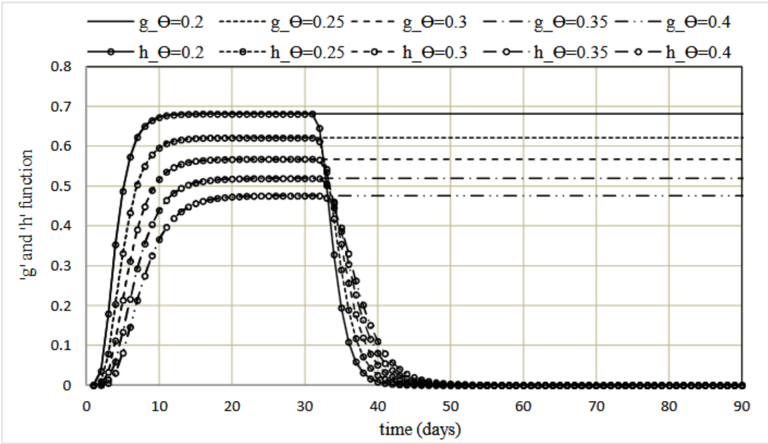

Figure 22. g_c and h_c functions of the well's concentration for different values of Θ

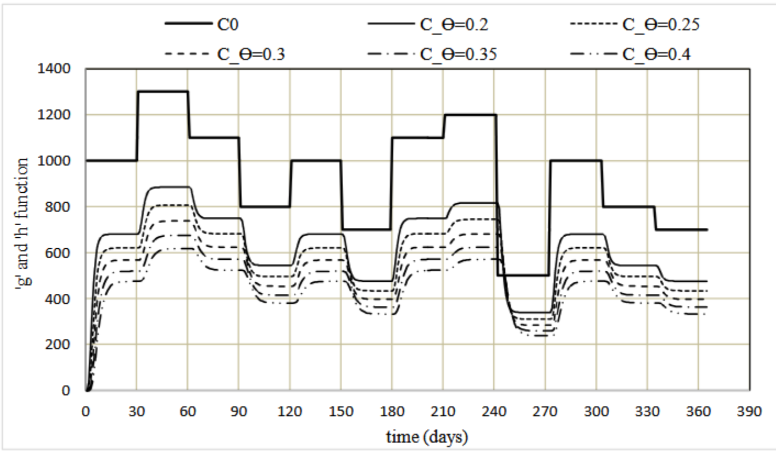

Figure 23. Variations of concentration in the well (*C(t)*) for different values of Θ

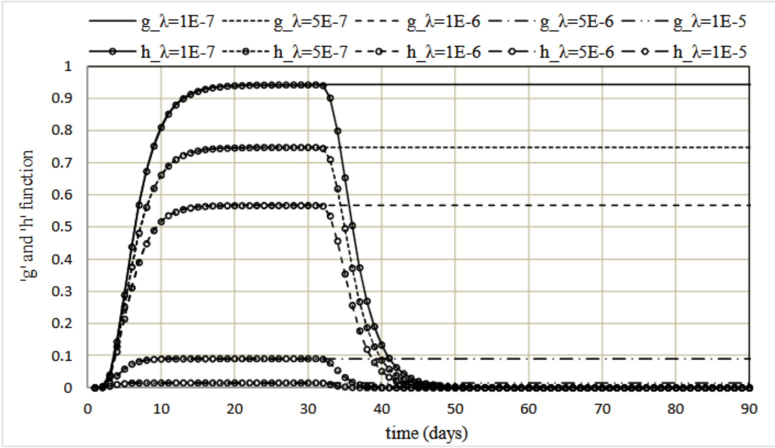

Figure 24. g_c **and** h_c **functions of the well's concentration for different values of** λ

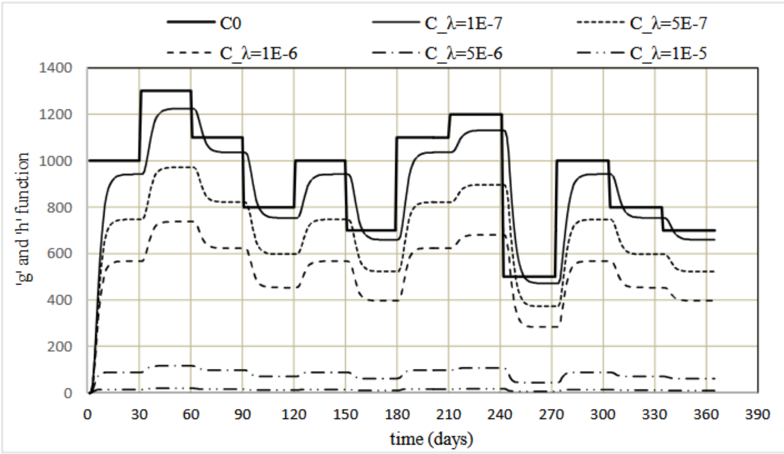

Figure 25. Variations of concentration in the well (*C(t)*) for different values of λ

5. Conclusion

Riverbank Filtration (RBF) systems utilize the natural filtration processes of riverbed and aquifer materials to purify water as it moves from a surface water source to extraction wells. The application of Linear Systems Response Functions in RBF systems enhances their design, management, and operational efficiency, particularly in modeling and predicting groundwater flow and contaminant transport under variable conditions.
Clearly, the novelty of this work is as follows:

Using the concept of response functions of linear systems in the modeling of contaminant transport in groundwater systems. Linear Systems Response Functions, such as unit pulse response functions, are used to model the drawdown of the groundwater table caused by pumping wells, and to predict the solute concentration extending in the aquifer.
In this paper, a comprehensive optimization model for the design of riverbank filtration systems has been developed with the aim of minimizing the total cost of the system, encompassing the locations and pumping rates of wells, as well as the dimensions of other system components. This method enables the analytical calculation of the optimal location and pumping rate of

wells in the RBF system. A simulation model was also developed for analyzing these systems. In these models, the analytic solutions of the equations of groundwater flow and pollutant transport were used.

The analysis indicated that:

- For $t>1$, the unit pulse response function of drawdown $h_w(t)$ is independent of the locations of wells and aquifer storativity.
- The transient flow equation in RBF systems reaches steady (pseudo-steady) conditions. The drawdown in variable pumping for a given transmissivity and distance of wells is only a function of the pumping pattern of wells.
- Solving the steady optimization problem shows that unexpectedly, the discharges of the central and side wells were greater than and smaller than those of other wells. As both pumping and conveyance costs have been considered here, the conveyance cost (which increases with the distance of wells) dominated the pumping cost (which drops with the distance of wells) here, suggesting that if only the pumping cost had been considered, opposite results can be expected.
- The sensitivity analysis revealed that *UCE* (unit cost of energy) is a relatively more sensitive and R_d is a relatively less sensitive parameter, but generally all five parameters are sensitive. For four system parameters (other than *UCE*), the sensitivity ranking was: transmissivity (T), decay rate (λ), porosity (Θ), and retardation factor (R_d) (implementing the transient model by changing these four parameters confirmed the above-mentioned result).
- The proposed simulation model formulations are useful for assessing the computability of the variations of solute concentration in RBF systems, and the effects of changing parameters in the solute concentration in steady-state and transient conditions.

6. References

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